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Applying MGARCH Models in Finance

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ABSTRACT – *In this paper we give literature review about application of multivariate GARCH (MGARCH) models in modern finance and economy. First, we will present basic concept of multivariate volatility (GARCH) modeling. MGARCH models specify equations for how the covariance moves over time and these models have been designed to model the conditional covariance matrix of multiple time series. Problems of portfolio Value-at-Risk (VaR) estimates, portfolio optimization, risk assessment, volatility transmitting, asset allocation, hedging in futures markets, pricing of assets and derivatives, CAPM betas require a multivariate framework, because all problems mentioned above require covariances as inputs. This implicates very wide application of MGARCH models. Additionally, in this paper we will also describe the leverage effect in multivariate GARCH models.*

KEY WORDS: *multivariate GARCH models, volatility, application of MGARCH models, leverage effect*

Introduction

The goal of this paper is to give overview about application of multivariate GARCH (MGARCH) models in finance. The application of MGARCH models is very wide. Some of typical applications are: portfolio optimization, pricing of assets and derivatives, computation of the value at risk (VaR), futures hedging, volatility transmitting and asset allocation. All aforementioned problems require covariances as inputs, so these problems require a multivariate framework [15]. It should be noted that in many financial econometric models the conditional-variance equation plays a major role. For example, the systematic risk as measured by beta depends on the (conditional) second moments of the asset returns, and so does the minimum-variance hedge ratio. Reliable estimates and inference of these quantities depend on well-specified conditional heteroscedasticity models [24], [15].

The paper is organized as follows. In Section 2, we present basic concept of multivariate volatility (GARCH) modeling. In Section 3, we give literature review about application of multivariate GARCH (MGARCH) models in different areas of finance. Section 4 presents the leverage effect in multivariate GARCH models. Finally, Section 5 concludes.

Basic concept of volatility modeling

Multivariate GARCH models specify equations for how the variances and covariances move over time. Modeling a covariance matrix is difficult because of the likely high dimensionality of the problem and the constraint that a covariance matrix must be positive definite [18]. The crucial stage in MGARCH modeling is to provide a realistic but parsimonious specification of the variance matrix ensuring its positivity. Obviously a disadvantage of the multivariate approach is that the number of parameters to be estimated in the GARCH equation increases rapidly, which limits the number of assets that can be included [9], [15]. In this section we give an overview of different types of MGARCH models. The models covered are vector ARCH model (VEC, initially due to Bollerslev, Engle and Wooldridge, 1988), diagonal VEC model (DVEC), the BEKK model (named after Baba, Engle, Kraft and Kroner), Constant Conditional Correlation Model (CCC, Bollerslev, 1990), Dynamic Conditional Correlation Model (DCC models of Tse and Tsui (2002) and Engle (2002)) [16].

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The VEC model

The general multivariate GARCH(p,q) model is given as:

$$VEC\left(\Sigma_t\right) = C + \sum_{i=1}^{q} A_i \cdot VEC\left(\varepsilon_{t-i}\varepsilon_{t-i}^{'}\right) + \sum_{j=1}^{p} B_j \cdot VEC\left(\Sigma_{t-j}\right), \tag{2.1}$$

where A_i and B_j are parameter matrices containing $(N^*)^2$ parameters [with $N^* = N(N+1)/2$], whereas the vector C contains N^* coefficients. VEC is the column-stacking operator. We will assume that all eigenvalues of the matrix $\sum_{i=1}^{q} A_i + \sum_{j=1}^{p} B_j$ have modulus smaller than one, in which case the vector process ε_t is covariance stationary with unconditional covariance matrix given by Σ_t, say [12], [16].

The diagonal VEC (DVEC) model

Under the diagonal *VEC* (DVEC) model, each variance-covariance term is postulated to follow a GARCH-type equation. The model can be written as follows [25, 7]:

$$\sigma_{ij,t} = c_{ij} + \sum_{h=1}^{p} a_{hij}\varepsilon_{t-h,i}\varepsilon_{t-h,j} + \sum_{h=1}^{q} b_{hij}\sigma_{t-h,ij} \qquad 1 \le i \le j \le k\,, \tag{2.2}$$

where c_{ij}, a_{hij} and b_{hij} are parameters. The DVEC multivariate GARCH model could also be expressed as an infinite order multivariate ARCH model, where the covariance is expressed as a geometrically declining weighted average of past cross products of unexpected returns, with recent observations carrying higher weights. Now, we will present the diagonal VEC model in the form:

$$\Sigma_t = C_0^* + \sum_{i=1}^{m} A_i^* \odot \left(\varepsilon_{t-i}\varepsilon_{t-i}^{'}\right) + \sum_{j=1}^{s} B_j^* \odot \Sigma_{t-j} \tag{2.3}$$

where m and s are non-negative integers, and $\odot$ denotes Hadamard product[2] (element by element matrix multiplication) [21]. Let us define the symmetric $N \times N$ matrices A_i^* and B_j^* as the matrices implied by the relations $A = diag\left[vec\left(A^*\right)\right]$[3] for and $B = diag\left[vec\left(B^*\right)\right]$ for and C_0^* as given by $C = vec\left(C_0^*\right)$ [4]. The model which is represented by Eq. (2.3) is DVEC(m,s) model [21], [16].

The BEKK model

The BEKK model is in form:

$$\Sigma_t = C_0 C_0^{'} + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ki}^{'}\varepsilon_{t-i}\varepsilon_{t-i}^{'}A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} B_{ki}^{'}\,\Sigma_{t-i}\,B_{ki}, \tag{2.4}$$

where C_0 is a lower triangular matrix and A_{ki} and B_{ki} are $N \times N$ parameter matrices. The BEKK representation in Eq. (2.4) is a special case of Eq. (2.1) [12]. Based on the symmetric parameterization of

[2] If X = (x_{ij}) and Y = (y_{ij}) are both $m \times n$ matrices, then $X \odot Y$ is the $m \times n$ matrix containing element wise products ($x_{ij}y_{ij}$) [4].

[3] If v is a vector of dimensions m then $diag(v) = I_m \times v$ [4].

the model, □$_t$ is almost surely positive definite provided that C_0C_0' is positive definite [21]. The necessary condition for the covariance stationarity of the BEKK model is having the eigenvalues, i.e. the characteristic roots of $\sum_{i=1}^{q}\sum_{k=1}^{K}(A_{ik}^{*}\otimes A_{ik}^{*})+\sum_{i=1}^{p}\sum_{k=1}^{K}(B_{ik}^{*}\otimes B_{ik}^{*})$ less than one in modulus [17], [16].

Constant Conditional Correlation (CCC) Model

Bollerslev (1990) suggested a multivariate GARCH model in which all conditional correlation are constant and the conditional variances are modeled by univariate GARCH models. This so-called the Constant Conditional Correlation Model (CCC) [25]. Thus the CCC model is given by

$$\sigma_{ii,t}=c_i+\sum_{h=1}^{p}a_{hi}\varepsilon_{t-h,i}^{2}+\sum_{h=1}^{q}b_{hi}\sigma_{t-h,i}\qquad i=1,\dots,k \tag{2.5}$$

$$\sigma_{ij,t}=\rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}}\qquad 1\le i<j\le k \tag{2.6}$$

$$\rho_t=\rho=\left[\rho_{ij}\right],\rho_{ii}=1 \qquad [2, 25]. \tag{2.7}$$

where □$_t$ is the $N\times N$ conditional correlation matrix of □$_t$ and □$_t$ is symmetric with unit diagonal elements. The dynamic of the covariances is determined only by the dynamics of the two conditional variances. There are $N\left(N-1\right)/2$ parameters in □□2], [16□.

Dynamic Conditional Correlation Model (DCC)

For *N*-dimensional returns, Tse and Tsui (2002) assume that the conditional correlation matrix □$_t$ follows the model (DCC_T(M)) [21]:

$$\rho_t=\left(1-\theta_1-\theta_2\right)\rho+\theta_1\psi_{t-1}+\theta_2\rho_{t-1} \tag{2.8}$$

$$\psi_{ij,t-1}=\frac{\sum_{m=1}^{M}e_{i,t-m}e_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M}e_{i,t-m}^{2}\right)\left(\sum_{m=1}^{M}e_{j,t-m}^{2}\right)}},\ e_{it}=\varepsilon_{it}/\sqrt{\sigma_{iit}} \tag{2.9}$$

where θ_1 and θ_2 are scalar parameters, $\theta_1,\theta_2>0$ and $\theta_1+\theta_2<1$. ρ is like in CCC, i.e. $N\times N$ positive-definite matrix with unit diagonal elements, and ψ_{t-1} is the $N\times N$ sample correlation matrix using shocks (ρρ◎ from ρ = *t - M, t – M + 1, ... , t – 1* for a prespecified *M*. A necessary condition to ensure positivity of ψ_{t-1} is that $M\ge N$. Notice that $\psi_{ii,t-1}$= 1 for each *i* by construction [21, 2]. ρ$_t$ is a weighted average of correlation matrices (ρ, ψ_{t-1}, ρ$_{t-1}$). Hence, ρ$_t$ > 0 if any of three components is greater than zero. If $\theta_1=\theta_2=0$, the CCC model is obtained. Hence one can test for CCC against DCC_T(M) [2]. Estimation of the two scalar parameters θ_1 and θ_2 requires special constraints to ensure positive definiteness of the correlation matrix. The choice of □and *M* deserves a careful investigation [21], [16].

Engle (2002) proposes the model (DCC_E(1,1)):

$$\rho_t=\left(diag\,Q_t\right)^{-1/2}Q_t\left(diag\,Q_t\right)^{-1/2} \tag{2.10}$$

where $Q_t = (q_{ij,t})$ is a $N \times N$ matrix, symmetric and positive, given by
$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 e_{t-1}e_{t-1}^{'} + \theta_2 Q_{t-1}$, where $e_t = (e_{1t}, e_{2t}, \ldots, e_{Nt})$' is the standardized innovation vector with elements $e_{it} = \varepsilon_{it} / \sqrt{\sigma_{iit}}$, $\bar{Q}$ is the unconditional covariance matrix of e_t, it is $N \times N$ matrix, symmetric and positive, and θ_1 and θ_2 are non-negative scalar parameters satisfying $0 < \theta_1 + \theta_2 < 1$, what implies that $Q_t > 0$ and $\psi_t > 0$ [21, 2]. Q_t is the covariance matrix of e_t, since q_{iit} is not equal to one by construction. If $\theta_1 = \theta_2 = 0$, and $\bar{q}_{ii} = 1$, the CCC model is obtained. Hence one can test for CCC against $\mathrm{DCC_E}(1,1)$. In both DCC models, all the conditional correlations have the same dynamics [2], [16].

The application of MGARCH models

The success of the autoregressive conditional heteroscedasticity (ARCH) model and the generalized ARCH (GARCH) model in capturing the time-varying variances of economic data in the univariate case has motivated many researchers to extend these models to the multivariate dimension [23]. While univariate descriptions are useful and important, problems of risk assessment, asset allocation, hedging in futures markets and options pricing require a multivariate framework, since high volatilities are often observed in the same time periods across different assets [27]. There are many examples in which empirical multivariate models of conditional heteroscedasticity can be used fruitfully. An illustrative list includes the following analyses: model of the changing variance structure in an exchange rate regime (Bollerslev, 1990), calculation of the optimal debt portfolio in multiple currencies (Kroner and Claessens, 1991), evaluation of the multiperiod hedge ratios of currency futures (Lien and Luo, 1994), examination of the international transmission of stock returns and volatility (Karolyi, 1995) and estimation of the optimal hedge ratio for stock index futures (Park and Switzer, 1995) [23].

C. M. Hafner and H. Herwartz (1998) introduced volatility impulse response functions (VIRF) for multivariate time series exhibiting conditional heteroskedasticity. For the general VEC representation of multivariate GARCH models they provided the analytic expressions for VIRF. The structural dependence of the conditional correlation was shown to depend crucially on the multivariate GARCH specification in use. Volatility shocks were highly persistent for bivariate exchange rate series. Using the BEKK model as parameterization device they found deviations from a restricted version of the so-called diagonal model to be significant [14]. These authors (2006) introduced a new concept of impulse response functions tracing the effects of independent shocks on volatility through time while avoiding typical orthogonalization and ordering problems. They applied their methodology to a bivariate system consisting of foreign exchange (FX) rate series, and they discussed various examples of historical shocks and their impacts on volatility. They found, among other things, considerably different interpretations using VIRF and conditional volatility profiles, respectively, for shocks affecting the volatilities in an asymmetric way, that is, a shock that perturbs one series but not the other [12]. Again, same authors (2004) used the general VEC representation of multivariate GARCH model. They proved sufficiency or necessity of particular parameter restrictions for noncausality in variance (linear causality in variance). They showed that a convenient alternative to residual based testing was to specify a multivariate volatility model, such as multivariate GARCH (or BEKK), and constructed a Wald test on noncausality in variance. The Wald test was shown to have superior power properties under a sequence of local alternatives. Furthermore, they showed by simulation that the Wald test is quite robust to misspecification of the order of the BEKK model, but that empirical power decreases substantially when asymmetries in volatility are ignored [17]. In their paper (1998) these authors employed bivariate GARCH in Mean time series models in order to explain excess return of 20 major German stocks in the light of the CAPM framework with time varying betas. The dependence of beta on news was characterized with respect to different sources (asset specific vs. market general news). The empirical result suggested that negative news emerging from the market involve a stronger impact on beta relative to positive news [13].

Wenling Yang, David E. Allen (2004) estimated futures hedge ratios from four alternative modeling frameworks, an Ordinary Least Squares (OLS)-based model, a Vector Autorregression (VAR) model, a Vector Error-Correction model (VECM) and a diagonal VEC (DVEC) multivariate GARCH model. They compared the hedging effectiveness of these hedge ratios. The multivariate GARCH time-varying hedge ratios outperform the other constant hedge ratios, and MGARCH hedge ratios provide the highest rate of return as well as the greatest portfolio risk reduction [28]. Bera, Garcia and Roh (1997) reported that the BEKK model did not perform well in the estimation of the optimal hedge ratios. Lien, Tse and Tsui (2001) reported difficulties in getting convergence when using the BEKK model to estimate the conditional-variance structure of spot and futures prices [22].

To capture the time-varying feature of conditional correlation between equities and exchange rates, Tong (1996) adopted the BEKK multivariate GARCH model. The BEKK algorithm permits time-variation in the conditional covariance while it ensures the condition of a positive-definite, variance-covariance matrix. But as for many currency markets, Sheedy (1998) found that the BEKK specifications would not be effective in eliminating the correlation structure [26].

Siu Pang Au-Yeung and Gerard Gannon (2002) applied BEKK-GARCH model with multiple switch points in the variance equations and they found to capture the structural changes that have taken place in the Hong Kong markets. They found the BEKK-GARCH (1,1) model with 3 switching points was superior to other models with less switching points. It was able to capture structural changes in the volatility structure of the HSI and HSIF. Hence, their results showed there are significant impacts on the informational efficiency in the stock and futures market following the policy changes [17].

Matteo Bonato (2006) combined the appealing properties of the stable Paretian distribution to model the heavy tails and the GARCH model to capture the phenomenon of the volatility clustering. A multivariate GARCH structure (diagonal BEKK and VEC) is then adopted to model the covariance matrix of the Gaussian vectors underlying the sub-Gaussian system. He was applied the model to daily U.S. stock returns [6].

Aslihan Altay-Salih, Mustafa C. Pinar and sven Leyffer (2003) proposed a constrained nonlinear programming view of multivariate GARCH volatility estimation models in financial econometrics. Their results demonstrate that constrained nonlinear programming (NLP) is a worthwhile exercise for GARCH models, especially for the bivariate and trivariate cases, as they offer a significant improvement in the quality of the solution of the optimization problem over diagonal VECH and BEKK representation of the multivariate GARCH model [1].

Rombouts and Verbeek (2004) analyzed optimal portfolio choice focusing explicitly on downside risk. In particular, they investigated the economic value of multivariate volatility models (DVEC, DCC models of Tse and Tsui (2002) and Engle (2002)) when optimal portfolios are constructed under a Value-at-Risk (VaR) constraint. Value-at-Risk defines the maximum expected loss on an investment over a specified horizon at a given confidence level, and is used by many banks and financial institutions as a key measure for market risk. They explored the usefulness of semi-parametric multivariate GARCH models for asset returns for evaluating the VaR of a portfolio. They also illustrated how such models can be used to determine an optimal portfolio that is based on maximizing expected returns subject to a downside risk constraint, measured by VaR. The advantage of the multivariate approach is that the VaR of any portfolio of assets can be determined from the GARCH estimates and the corresponding non-parametric estimate of the multivariate distribution of the innovations [19].

C. Brooks, A.D. Clare and G. Persand (2002) have investigated the possible use of multivariate GARCH models with time-varying conditional covariances and correlations in calculating minimum capital risk requirements (MCRRs) for portfolios of assets. Their most important result is that more accurate MCRRs estimates might be achievable if they can consider the VaR problem in a multivariate context. The model that they have applied in paper appears to have been relatively accurate, at least compared with equivalent calculations based on univariate models [8].

Kroner and Sultan (1991) applied the constant-correlation bivariate GARCH (bivariate CCC-GARCH) model to hedge the currency exposure risk. While conditional variance of different assets and currency forward prices changes over time, the conditional correlations for currency markets are assumed to be constant in order to get a positive definite variance-covariance matrix as proposed in Bollerslev (1990). This constant-correlation approach has been widely applied because of its computational simplicity. But the financial data of equities and exchange rates provided strong evidence that the assumption of having a constant correlation was violated for these markets [26].

Luc Bauwens and Sebastien Laurent (2004) showed that a high dimensional VaR problem could be solved without imposing strong restrictions on both the covariance structure and the distribution of the innovations. They showed that new family of distributions (the multivariate skew-Student density) combined with a multivariate DCC-GARCH model is useful for modeling financial returns and forecasting the Value-at-Risk (VaR) of portfolios of assets. Indeed, they found that the multivariate skew-Student density provides better, or at least not worse, out-of-sample VaR forecasts than a symmetric density [3].

Robert Engle (2002) showed in his paper that the bivariate version of DCC model provides a very good approximation to a variety of time varying correlation processes. The comparison of DCC with simple multivariate GARCH and several other estimators showed that the DCC is often the most accurate. This is true whether the criterion is mean absolute error, diagnostic tests or tests based on value at risk calculations. Empirical examples from typical financial applications are quite encouraging as they reveal important time varying features that might otherwise be difficult to quantify. Statistical tests on real data indicated that all of tested models are misspecified but that the DCC models are competitive with the multivariate GARCH specifications and superior to moving average methods [11].

Shohreh Valiani (2004) adopted MGARCH specification that has been applied to estimate the time-varying correlations of underlying assets and related currency forwards in order to hedge the currency exposure risk in an international portfolio context. His paper adopted a simple conditional risk minimizing model and based on the time-varying correlations of security and currency forward exchange rate returns estimate the optimal weights of investment in different underlying assets accompanied by the necessary amounts of currency forward hedges. The empirical investigation shows that the optimal multivariate GARCH dynamic hedging strategy can capture the currency fluctuations the best and over-performs the risk controlling procedure [26].

John Elder (2003) derived an analytical expression for an impulse-response function for a vector autoregression with multivariate GARCH errors, where the vector of conditional means is a function of the conditional variances. He also provided the appropriate interpretation of an impulse-response function for such models and suggest interesting empirical issues that can be addressed within this framework [10].

Y. K. Tse and Albert K. C. Tsui (1998), applied the multivariate GARCH model with time-varying correlation (DVEC) to three data sets, namely, the exchange rate data, the national stock market data and the sectoral price data. This model was found to pass the model diagnostics satisfactorily and compared favorably against the BEKK model, while the constant-correlation MGARCH (CCC) model was found to be inadequate. Extending the constant-correlation model to allow for time-varying correlations provided some interesting empirical results. In particular, the estimated conditional-correlation path provides an interesting time history that would not be available in a constant-correlation model [22].

Tim Bollerslev (1989) proposed a multivariate time series model with time varying conditional variances and covariances, but constant conditional correlations (CCC model). Parametrizing each of the conditional variances as a univariate GARCH process, the descriptive validity of the model was illustrated for a set of five nominal European U.S. dollar exchange rates following the inception of the European Monetary System (EMS). When compared to the pre-EMS free float period, the comovements between the currencies were found to be significantly higher over the later period [5].

Michael Schröder and Martin Schüler (2003) attempted to assess the Europe-wide systemic risk in banking. Systemic risk is one of the main reasons why banks are regulated and supervised. As a measure of systemic risk they used the conditional correlations between pairs of national bank stock indices of the EU countries. The correlations were estimated using bivariate GARCH-models which considered the influence of the national stock market index and a short-term interest rate as explanatory factors. The correlations measured the linear relationships between the residuals of the GARCH-models and as these residuals mainly reflect bank specific factors they are suitable to quantify the systemic risk [20].

Jelena Minović (2007) applied MGARCH (restricted BEKK, DVEC and CCC) models to the analysis of the Serbian financial market. She considered bivariate and trivariate time series models. The main finding of her paper is that conditional covariances exhibited significant changes over time for the both stocks (Hemofarm and Energoprojekt) and the index (BELEX15). So, the correlation between log returns of stocks and index was very unstable over time in Serbian frontier markets. She showed that even restricted version of BEKK, DVEC and CCC models with reduced number of parameters could gave fairly accurate results (for detail see reference [15]).

Leverage effect in multivariate GARCH models

Leverage effect is the tendency for volatility to rise more following a large price fall than following a price rise of the same magnitude [7]. For example, for stock returns, negative shock may have a larger impact on their volatility than positive shocks of the same absolute value (this effect is unveiled by Black 1976). In other words, the news impact curve, which traces the relation between volatility and the previous shock, is asymmetric. For multivariate series the same argument applies: the variances and the covariances may react differently to a positive than to a negative shock [4]. Most multivariate GARCH models do not allow for asymmetries [9].

Kroner and Ng (1998) applied the model to large and small firm returns. They found that bad news about large firms can cause additional volatility in both small-firm and large-firm returns. Furthermore, this bad news increase the conditional covariance. Small firm news had only minimal effects [4].

Peter De Goeij and Wessel Marquering (2004) analyzed bond and stock market interactions by modeling the time-varying covariances between stock and bond market returns. The main contribution of their paper is that it extends the multivariate model by allowing for asymmetric effects in covariances between stock and bond returns. They used the asymmetric diagonal VECH[4] model in the matrix notation:

$$\Sigma_{t+1} = C + B \odot \Sigma_t + A_1 \odot \varepsilon_t \varepsilon_t^{'} + A_2 \odot \left(\varepsilon_t^{-} \varepsilon_t^{-'}\right) + A_3 \odot \left(\varepsilon_t^{-} \varepsilon_t^{+'}\right) + A_4 \odot \Im\left(\varepsilon_t^{+} \varepsilon_t^{-'}\right),$$

where C, B, A_1, A_2, A_3, and A_4 are ($N \times N$) parameter matrices, $\Im$ is the operator that permutes rows of a square matrix, in such a way that the lower triangular part of the matrix is substituted by the upper triangular part of the matrix, and $\varepsilon_t^{-} = \left[I_{\varepsilon_{1,t}} \varepsilon_{1,t}, \ldots, I_{\varepsilon_{N,t}} \varepsilon_{N,t} \right]^{'}$ and

$\varepsilon_t^{+} = \left[\left(1 - I_{\varepsilon_{1,t}}\right) \varepsilon_{1,t}, \ldots, \left(1 - I_{\varepsilon_{N,t}}\right) \varepsilon_{N,t} \right]^{'}$ [9].

They showed that asymmetric effects are present in the covariances between stock returns and returns on a second asset. They also showed that daily returns on the S&P 500 and NASDAQ indexes exhibit significant leverage effects. Not only variances, but also covariances, between stock and bond returns exhibit significant asymmetries. Overall their findings imply that a symmetric specification is too restrictive to model the conditional covariances. Especially bad news in the stock market is followed by a much higher conditional covariance than good news in the stock market. This holds irrespective of the sign of the bond market shock. The cross effects in asymmetries appear to be important as well. Covariance between stock and bond returns tend to be relatively low after bad news in the stock mar-

[4] VECH is the operator that stacks the lower triangle of a $N \times N$ matrix as an $N^* \times 1$, $N^*=N(N+1)/2$ vector, while VEC is the operator that stacks a matrix as a column vector [15].

ket and good news in the bond market. Thus they find evidence that the cross-asymmetry terms are important when modeling covariances between asset returns. Overall the results indicate that the performance of the asymmetric diagonal VECH model[5] of conditional second moments is quite good. Asymmetries in covariances have important implications for portfolio managers [9].

Peter De Goeij and Wessel Marquering inferred that the conditional covariance is high after shocks of the same sign, while shocks in opposite directions lower the conditional covariance. This is because bond returns are positively correlated. As the assets move together, shocks in the same direction involve a higher risk than shocks in opposite directions. This make sense, as it is riskier to invest in two assets that are highly positively correlated than to invest in two assets that are less correlated [9].

Hansson and Hordahl (1998) add the term $D \odot v_{t-1}v_{t-1}^{'}$, in a DVEC model like (2.3). Where D is a diagonal matrix of parameters. To incorporate the leverage effect in the (bivariate) BEKK model, Hafner and Herwartz (1998) add the terms $D_1^{'}\varepsilon_{t-1}\varepsilon_{t-1}^{'}D_1 1_{\{\varepsilon_{1,t-1}<0\}} + D_2^{'}\varepsilon_{t-1}\varepsilon_{t-1}^{'}D_2 1_{\{\varepsilon_{2,t-1}<0\}}$, where D_1 and D_2 are 2×2 matrices of parameters and $1_{\{...\}}$ is the indicator function. This generalizes the univariate GJR specification (for detail see reference [15]) [4].

Conclusion

The main aim of this paper is application MGARCH models in different area economy and finance. We present some literature review about application of these models. In the beginning of this paper we introduced basic concept of multivariate volatility (GARCH) modeling. The key is that in many financial econometric models the conditional-variance and conditional-covariance equations plays a major role. There are many examples in which empirical multivariate models of conditional heteroscedasticity can be used fruitfully. So, multivariate descriptions are useful and important, because problems of risk assessment, asset allocation, hedging in futures markets and options pricing, portfolio Value at Risk estimates, CAPM betas, require a multivariate framework and covariances as inputs.

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