mathematics

Review

Sectoral Efficiency and Resilience: A Multifaceted Analysis of S&P Global BMI Indices Under Global Crises

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Abstract: This study investigates the complexity, efficiency, and sectoral interdependencies of the S&P Global BMI indices during critical global events, including the COVID-19 pandemic and the Russia–Ukraine war. The analysis is conducted in three dimensions: (1) evaluating market efficiency using permutation entropy and the Fisher information measure, (2) exploring sectoral alignments through clustering techniques (hierarchical and k-means clustering), and (3) assessing the influence of geopolitical risk using Multifractal Detrended Cross-Correlation Analysis (MFDCCA). The results highlight significant variations in informational efficiency across sectors, with Utilities and Consumer Staples exhibiting high efficiency, while Emerging Markets and Financials reflect lower efficiency levels. Temporal analysis reveals widespread efficiency declines during the pandemic, followed by mixed recovery patterns during the Ukraine conflict. Clustering analysis uncovers dynamic shifts in sectoral relationships, emphasizing the resilience of defensive sectors and the unique behavior of Developed BMI throughout crises. MFDCCA further demonstrates the multifractality in cross-correlations with geopolitical risk, with Consumer Staples and Energy showing stable persistence and Information Technology exhibiting sensitive complexity. These findings emphasize the adaptive nature of global markets in response to systemic and geopolitical shocks, offering insights for risk management and investment strategies.

Keywords: S&P Global BMI sectoral indices; price dynamics; efficient market hypothesis; informational efficiency; clustering; MFDCCA; COVID-19; Russian–Ukraine war; geopolitical risk

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1. Introduction

Financial markets are complex and dynamic systems shaped by a wide range of factors, including economic conditions, geopolitical events, investor sentiment, and market structures [1–3]. Recent global crises, notably the COVID-19 pandemic and the Russia–Ukraine war, have amplified market volatility, disrupted sectoral relationships, and triggered significant behavioral shifts among investors [4,5]. Financial markets are increasingly influenced by global crises, with significant implications for economic stability and investment strategies. Understanding how sectoral efficiencies and interdependencies respond to geopolitical risks is crucial for improving risk management practices and maintaining resilient financial systems.

Market efficiency, as posited by the Efficient Market Hypothesis (EMH), suggests that asset prices fully reflect all available information, thereby implying that geopolitical risks are rapidly incorporated into market valuations. However, the degree of efficiency can vary across sectors due to their unique exposure and sensitivity to geopolitical events. Sectoral interdependencies further complicate this relationship, as shocks in one sector can propagate through supply chains, financial linkages, and investor sentiment, amplifying or mitigating the overall impact of geopolitical risks. By integrating these theoretical frameworks, this study aims to elucidate how market efficiency and sectoral interconnectedness shape the transmission and pricing of geopolitical risk, offering a more structured and comprehensive perspective on their interplay.

Prior studies have shown that stock indices exhibit heterogeneous behavior during crises, with interdependency variations across markets and asset classes [6,7]. Defensive sectors, such as Utilities and Consumer Staples, often display heightened resilience during periods of market turbulence, while cyclical sectors, like Financials and Energy, are more susceptible to systemic shocks [8,9]. Moreover, recent research highlights the intricate relationship between geopolitical risk and financial market dynamics, emphasizing the nonlinear interactions and cross-correlations across indices during crises [10]. Such findings align with broader analyses that stress the adaptive nature of global markets in response to systemic shocks [11,12]. For example, the onset of the COVID-19 pandemic triggered widespread declines in efficiency across global stock markets, with financial and consumer-oriented sectors experiencing pronounced disruptions due to heightened volatility and shifts in investor behavior [13,14]. Similarly, the Russia–Ukraine conflict introduced volatility, particularly within energy markets, highlighting the sensitivity of certain sectors to geopolitical tensions [15–17].

The theoretical foundations of market behavior during crises emerge from three interconnected domains. First, market efficiency theory suggests that markets' capacity to process information varies systematically across sectors and crisis types [11,12]. This variation manifests through differential efficiency patterns, with some sectors maintaining robust information processing capabilities while others experience significant disruptions [13,14]. Second, sectoral interdependence theory examines how market relationships evolve during crises, with evidence showing that sectoral connections intensify and reshape during periods of stress [8–10]. Third, geopolitical risk transmission theory explains how political tensions affect markets through both direct channels, such as trade disruptions, and indirect channels, including investor sentiment shifts [15–17].

To examine these theoretical dimensions empirically, we employ three complementary methodological approaches. Our efficiency analysis utilizes permutation entropy and Fisher information measures [18,19], enabling precise quantification of how different sectors maintain or lose efficiency during crises. Our investigation of sectoral relationships employs advanced clustering techniques [19], revealing how market structures evolve under stress. Our analysis of geopolitical risk transmission uses multifractal methods [20], capturing the complex, scale-dependent nature of risk propagation.

We examine these dynamics using the S&P Global BMI sectoral indices, which offer a comprehensive framework for analyzing global equity market performance across distinct economic sectors. Derived from the S&P Global Broad Market Index (BMI)—which encompasses over 14,000 constituents from both developed and emerging markets—these indices are classified according to the Global Industry Classification Standard (GICS) [21]. The GICS structure includes 11 sectors: Communication Services, Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Real Estate, and Utilities. This classification provides a robust foundation for sector-specific benchmarking and facilitates nuanced analysis of global market trends [22]. In addition to the 11 main sectors, this study incorporates indices such as the S&P Developed BMI, S&P Emerging BMI, S&P Global BMI Gold, and S&P Global BMI, which are essential for capturing regional market variations, sector-specific trends, and broader global market performance [23].

This paper makes three key contributions. First, we demonstrate significant variations in sectoral efficiencies, showing that Utilities and Consumer Staples consistently exhibit higher efficiency, while Emerging Markets and Financials reflect lower efficiency levels. Consumer Staples experienced efficiency declines during the COVID-19 pandemic, while Utilities and Communication Services showed recovery during the Russia–Ukraine conflict, highlighting sector-specific adaptability. Second, we uncover dynamic shifts in sectoral relationships, emphasizing the resilience of defensive sectors and the unique adaptability of Developed BMI indices during systemic shocks. Clustering analysis reveals the distinct positioning of Developed BMI as a global benchmark and the formation of resilient clusters dominated by essential sectors during crises. Third, we highlight the multifaceted influence of geopolitical risk on market dynamics, with evidence of stable persistence in the Consumer Staples and Energy sectors, contrasted by heightened complexity in Information Technology. Sectors such as Communication Services and Energy demonstrated pronounced sensitivity to large-scale geopolitical shocks, emphasizing the need for adaptive risk management strategies.

The remainder of this paper is structured as follows: Section 2 outlines the methodology employed in the analysis, and Section 3 summarizes the data used. Section 4 presents and discusses the results, including clustering patterns, market efficiency dynamics, and the relationship with geopolitical risk. Finally, Sections 5 and 6 provide discussion and conclude the study, summarizing key findings and providing insights for risk management and investment strategies.

2. Materials and Methods

Before detailing specific methodological approaches, we establish how our analytical framework operationalizes the theoretical constructs discussed in the Introduction. Our methodology examines market behavior through three complementary approaches, each addressing a distinct aspect of financial market dynamics during crisis periods. The first approach employs information-theoretical measures to quantify market efficiency. These measures assess how effectively markets incorporate new information, testing the adaptive market efficiency hypothesis during periods of stress. The second approach utilizes clustering techniques to reveal evolving sectoral relationships during market stress. This analysis maps changing interdependencies and their implications for portfolio diversification strategies. While hierarchical clustering reveals nested structures without imposing strict assumptions about cluster shapes, k-means clustering identifies distinct market segments through variance minimization. The third approach applies MFDCCA to capture the complex, scale-dependent nature of geopolitical risk transmission. This step reveals the multifractal nature of cross-correlations, uncovering how geopolitical risk influences the persistence, complexity, and fluctuations of financial indices across different time periods.

This integrated framework examines the S&P Global BMI sectoral indices across three critical periods: pre-COVID-19, during COVID-19, and during the Russia–Ukraine war. The inclusion of the Geopolitical Risk (GPR) index enables assessment of how geopolitical uncertainties shape market dynamics. By combining these complementary methods, we capture both linear and nonlinear relationships, as well as time-varying dynamics across different scales.

2.1. Complexity and Efficiency Measures

Following the approach given by [18–20,24], we first explore the interplay between entropy (disorder), predictability, and informational efficiency using three primary components: permutation entropy, the Fisher information measure, and a sliding window approach.

Permutation entropy is employed to quantify the level of complexity or randomness within a time series. This approach identifies ordinal patterns inherent in the data by mapping symbolic sequences to specific data segments. Using these sequences, a probability distribution function is constructed to evaluate the complexity of the series [18,19,24]. For a time series of length Q, overlapping segments of length d, $Z_q = \left(z_q, z_{q+1}, \ldots, z_{q+d-1}\right)$ are analyzed, with the sequence ranking based on ascending values. The measure computes the permutation entropy as $H(d) = -\sum_{\pi} \pi log p(\pi)$, where $p(\pi)$ denotes the probability of each permutation [24]. Following standard practice, the embedding dimension d is selected to ensure statistical robustness, typically adhering to n>5d! [24]. Then, we utilized the Fisher information measure (FIM) to quantify the degree of order or indeterminacy within the system. This metric reveals insight into the system's disorder and provides a measure of the information extractable from the data [25,26]. The discrete normalized form of FIM, expressed as $F[P] = F_0 \sum_{i=1}^{N} \left(\sqrt{p_{i+1}} - \sqrt{p_i} \right)^2$, where p_i and p_{i+1} represent consecutive probabilities from P and F_0 is a normalization constant, is applied to evaluate the system's characteristics [19].

To perform a time-dependent analysis of PE and FIM, we applied the sliding window technique [19]. This method involves constructing a sequence of overlapping windows from the original time series $x_1, x_2, \ldots, x_N$. Specifically, the sliding windows $k_t x_{1+t\Delta}, \ldots, k_t x_{w+t\Delta}$ are generated for $t = 0, 1, \ldots, \left[\frac{N-w}{\Delta}\right]$, where w is the window size, Δ is the sliding step, and [] represents the integer part of a value. For each window, the corresponding permutation entropy and FIM values are calculated. This process enables the analysis of the dynamic behavior of the time series, capturing the temporal evolution of its complexity and informational efficiency. The permutation entropy and FIM are then mapped onto a Shannon–Fisher causality plane (SFCP), providing a comprehensive view of the dynamics. The results are further visualized through their trajectory in the SFCP, providing insights into changes in the system's dynamics over time.

Compared to traditional market efficiency measures like variance ratios or autocorrelation tests widely used in financial practice, PE and FIM can better detect nonlinear patterns in market behavior. However, these information-theoretic measures may be more computationally intensive than standard linear approaches.

2.2. Clustering Analysis

The second methodological step is to perform cluster analysis to identify patterns and similarities within the S&P Global BMI sectoral indices. Two primary approaches are employed—k-means clustering [27] and hierarchical clustering [28]. The k-means algorithm distributes data points into k clusters, where the centroid of each cluster is iteratively recalculated to optimize the grouping. The algorithm assigns each data point to the nearest cluster based on minimized distances, repeating the process until the convergence criteria are met. The optimal number of clusters, k, is determined using the elbow method, which evaluates the variance within clusters to identify the point of diminishing returns in increasing k [29]. Complementary to k-means, hierarchical clustering examines the inherent similarity patterns in the dataset. This method calculates pairwise dissimilarities between data points, constructing a hierarchical tree or dendrogram representing nested groupings. Each data point is initially treated as an individual cluster, and pairs of clusters are merged iteratively based on the least dissimilarity, creating a hierarchy. This iterative process continues until all data points are combined into a single cluster. The resultant dendrogram is analyzed to determine the most significant clusters, ensuring they align with observed patterns. These clustering methods are applied to the normalized data [19], comprehensively analyzing behavioral similarities across the dataset.

The selection of both hierarchical and k-means clustering methods is motivated by their complementary analytical strengths in capturing market relationships. Rather than claiming superiority over a single-method technique, we use both methods to provide different perspectives on sectoral relationships during crisis periods. While hierarchical clustering reveals nested structures of sectoral relationships without imposing strict assumptions about cluster shapes, k-means clustering offers advantages in identifying distinct market segments through variance minimization. Data normalization using z-score standardization prevents scale differences from biasing clustering outcomes.

2.3. Multifractal Detrended Cross-Correlation Analysis (MFDCCA)

Finally, the study includes the GPR index to capture the influence of geopolitical events on market volatility and sectoral interdependencies. We employ the Multifractal Detrended Cross-Correlation Analysis (MFDCCA) method to investigate multifractal properties between two non-stationary time series, see [30]. The steps in MFDCCA include the following.

The inputs for the analysis are two time series, x and y, of the same length N.

Step 1. Profile Construction

For two given time series, the respective profiles are created by integrating deviations from their mean values:

$$X(t) = \sum_{i=1}^{t} (x(i) - \overline{x}), \qquad Y(t) = \sum_{i=1}^{t} (y(i) - \overline{y}),$$

where $\overline{x}$ and $\overline{y}$ represent the mean of each series and where $t = 1, \ldots\ N$.

Step 2. Segmentation

Both profiles are divided into $N_s = [N/s]$ non-overlapping segments of equal length s, where s is the scale parameter. To maximize data usage, segmentation is performed from both ends of the time series.

Step 3. Detrending

For each segment, local trends are calculated using a least-squares polynomial fit. The detrended profiles are then determined by subtracting the local trends from the data in each segment.

Step 4. Cross-Covariance Calculation

The detrended covariance for each segment is computed as follows:

$$F^2(s,\ v) = \frac{1}{s} \textstyle\sum_{i=1}^{s} [X(v-1)s + i) - \widetilde{X}_v(i)][Y(v-1)s + i) - \widetilde{Y}_v(i)],$$

where $\widetilde{X}_v(i)$ and $\widetilde{Y}_v(i)$ are the polynomial fits for the respective segments.

Step 5. Fluctuation Function Calculation

The fluctuation function, averaged across all segments, is calculated for a range of orders q:

$$F_q(s) = [\frac{1}{2N_s} \sum_{v=1}^{2N_s} \left(F^2(s,\ v)\right)^{\frac{q}{2}}]^{1/q}.$$

For $q = 0$, logarithmic averaging is used:

$$F_q(s) = exp[\frac{1}{2N_s}\sum_{v=1}^{2N_s} lnF^2(s,\ v)\ ].$$

Then, we estimate the generalized Hurst exponent $h_{xy}(q)$ through the MFDCCA process, as outlined in Steps 1–5. Specifically, the fluctuation function is calculated for a range of scales s and orders q. The relationship between the fluctuation function $F_q(s)$ and the scale s is examined on a log–log plot. The generalized Hurst exponent $h_{xy}(q)$ is estimated as the slope of the relationship $F_q(s) \propto\ s^{h_{xy}(q)}$. The generalized Hurst exponent $h_{xy}(q)$ is a key component of MFDCCA and is used to characterize the scaling behavior and multifractality of cross-correlations between two time series. Unlike the traditional Hurst exponent, which is often associated with fractional Brownian motion and measures the long-range dependence or autocorrelation structure of a single time series, the generalized Hurst exponent captures the multifractal nature of cross-correlations across different scales. In this study, we use it to analyze how the cross-correlations between the GPR index and the S&P Global BMI sectoral indices vary with the scale of fluctuations, providing insights into the multifractal dynamics of these relationships.

A larger range of α and significant variations in $h_{xy}(q)$ indicate strong multifractality and complexity in the cross-correlations between the two time series. Persistent cross-correlations ($h_{xy}(2) > 0.5$) suggest a positive relationship, while anti-persistent behavior ($h_{xy}(2) < 0.\ 5$) reflects a negative relationship. The degree of multifractality is determined by calculating the range of $h_{xy}(2)$, where a larger $\Delta H = h_{xy}(q_{min}) - h_{xy}(q_{max})$ indicates a more pronounced multifractal characteristic. Using the generalized Hurst exponent, the multifractal mass exponent $\tau(q)$ is calculated:

$$\tau(q) = qh_{xy}(q) - 1.$$

The singularity strength, α_{xy}, represents the degree of singularity for each segment within a complex system, while the singularity spectrum, $f_{xy}(\alpha)$, describes the fractal dimension corresponding to α_{xy}. These measures are calculated using the following relation:

$$\alpha_{xy} = h_{xy}(q) + qh'_{xy}(q),$$

where $h'_{xy}(q)$ is the derivative of $h_{xy}(q)$ with respect to q. The singularity spectrum $f(\alpha)$, which describes the fractal dimensions of subsets, is obtained as $f\left(\alpha_{xy}\right) = q\alpha_{xy} - \tau(q)$.

The range of the singularity strength, $W = \alpha_{xy,max} - \alpha_{xy,min}$, quantifies the degree of multifractality. A larger W indicates stronger multifractal properties, reflecting more intense fluctuations within the system. The multifractal spectrum $f\left(\alpha_{xy}\right)$ is quantitatively characterized by key parameters that describe its width and asymmetry. The skew parameter $r = \frac{\alpha_{xy,max} - \alpha_0}{\alpha_{xy,min} - \alpha_0}$ evaluates the asymmetry of the spectrum, where α_0 is the overall Hurst exponent. A symmetric spectrum corresponds to $r = 1$, a right-skewed spectrum to $r > 1$ (dominated by small fluctuations), and a left-skewed spectrum to $r < 1$ (dominated by large fluctuations). The parameters W, α_0, and r are integral in assessing complexity. A more complex time series is characterized by a higher α_0, a broader spectrum width W, and a right-skewed shape ($r > 1$). In contrast, a narrower W, lower α_0, and left-skewed shape ($r < 1$) suggest reduced complexity. These measures provide a robust framework for evaluating the intricate dynamics and persistence of multifractal systems.

Compared to traditional bivariate analysis methods, MFDCCA combines the multifractal spectrum analysis with cross-correlation capabilities, allowing it to capture complex relationships between two time series across multiple scales. This method is particularly powerful for analyzing financial markets as it can simultaneously examine scaling heterogeneity and interdependence between series, revealing how market relationships vary across different time scales. While MFDCCA effectively characterizes the multifractal nature of cross-correlations, it cannot determine the directional causality in the relationships between the analyzed series.

In summary, our methodological framework combines three complementary approaches that offer several advantages over traditional methods. The permutation entropy and Fisher information measures can detect both linear and nonlinear patterns in market behavior, extending beyond conventional efficiency measures like variance ratios that primarily capture linear dependencies. The dual clustering approach provides richer insights into market structures compared to single-method clustering, though it requires assumptions about cluster shapes and distances. The MFDCCA framework captures scale-dependent relationships between markets and geopolitical risk, offering deeper insights than standard correlation analysis, while requiring larger datasets for reliable estimation. While alternative approaches such as wavelet analysis or dynamic copulas could provide different perspectives, our chosen methods balance analytical depth with interpretability, particularly important when examining market behavior during crisis periods.

3. Data

This study analyzes the daily closing prices of 15 S&P Global BMI sectoral indices, sourced from the S&P Dow Jones Indices, a division of S&P Global (www.spglobal.com/spdji, accessed on 17 October 2024), from 30 September 2014 to 16 October 2024, encompassing 3664 observations. It examines three key periods—pre-crisis (before 24 February 2020), the COVID-19 pandemic (24 February 2020–31 December 2021), and the period following its resolution, which overlaps with the ongoing Russia–Ukraine war (from 24 February 2022 onward)—to assess how global crises impact sectoral efficiency and resilience.

The indices represent a diverse range of sectors, including Developed Markets (A), Emerging Markets (B), Communication Services (C), Consumer Discretionary (D), Consumer Staples (E), Energy (F), Financials (G), Gold (H), Health Care (I), Industrials (J), Information Technology (L), Materials (M), Real Estate (N), Utilities (O), and Global (P). The analysis period encompasses major global events, including the COVID-19 pandemic and the Russia–Ukraine War, allowing for the exploration of how extreme shocks impact market efficiency and sectoral behavior.

The S&P Global BMI index structure means some companies are represented in both sectoral- and market-level indices. This does not weaken the analysis; rather, it provides a more complete view of market behavior. Sectoral indices highlight industry-specific trends, while developed and emerging market indices reflect differences in market structure and risk exposure. Including both shows how different market segments interact and respond to crises, offering deeper insight into sectoral resilience and overall market dynamics.

The descriptive statistics reveal several key patterns in the S&P Global BMI sector returns. Table 1 provides descriptive statistics of the returns, including the mean, standard deviation, skewness, and kurtosis for each sector. The mean daily returns across sectors range from 0.00080% to 0.00288%, with Information Technology (L) showing the highest mean return and Emerging BMI (B) the lowest. All sectors exhibit positive skewness, indicating a tendency toward positive returns, with Real Estate (N) showing the most pronounced right-skewed distribution. The kurtosis values are consistently high across all sectors (ranging from 3.57 to 27.94), with Real Estate (N) displaying the highest kurtosis, suggesting frequent extreme returns and heavy-tailed distributions. The standard deviations vary from 0.003034 to 0.008295, with Gold (H) showing the highest volatility and Consumer Staples (E) the lowest, reflecting different levels of risk across sectors.

The time series of daily closing prices for the S&P Global BMI sectoral indices, presented in Figure 1, illustrates the overall trends and variability across sectors from 2014 to 2024. Additionally, the histograms in Figure 2 provide a detailed distribution of these prices, highlighting the statistical characteristics and frequency of values for each index.

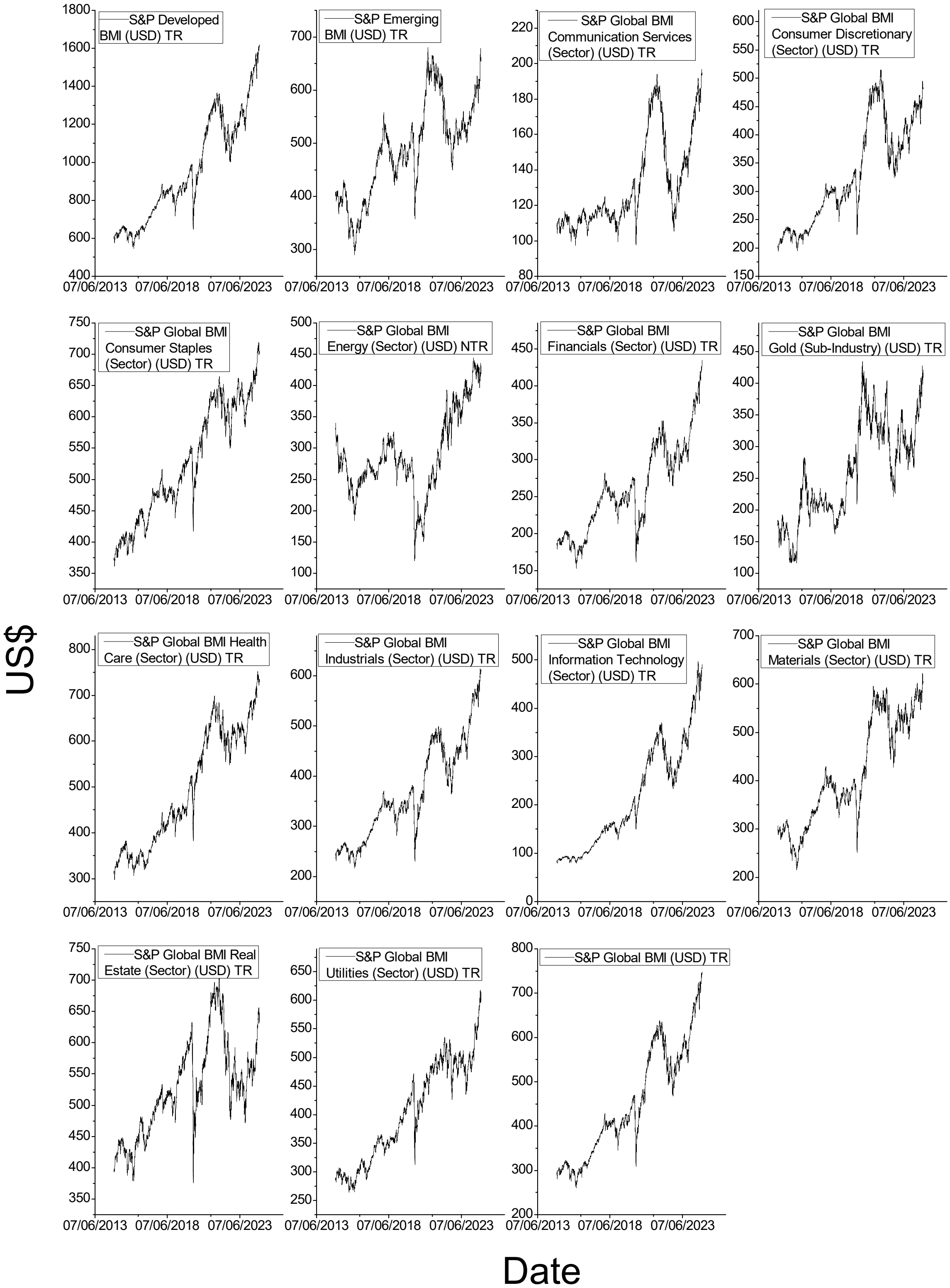


Figure 1. Time series of daily closing prices for S&P Global BMI sectoral indices (2014–2024).

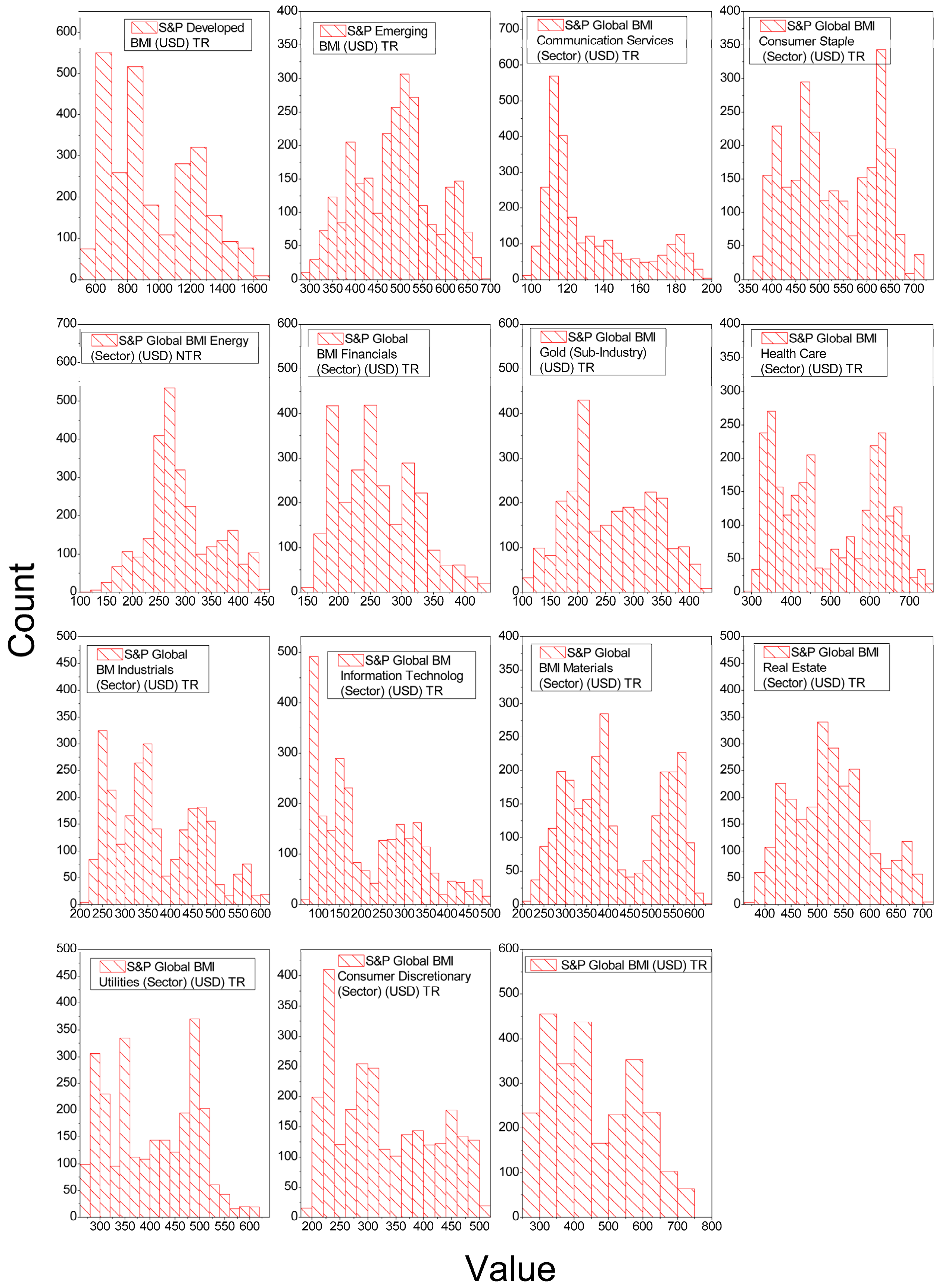


Figure 2. Histogram of daily closing prices for S&P Global BMI sectoral indices (2014–2024).

Table 1. Descriptive statistics of the returns.

Full Name	Code	Mean	Std	Skew	Kurtosis
S&P Developed BMI (USD) TR	A	0.000160	0.004065	1.127797	17.52093
S&P Emerging BMI (USD) TR	B	0.000080	0.004085	0.729691	6.020300
S&P Global BMI Communication Services (Sector) (USD) TR	C	0.000094	0.004371	0.626321	7.745967
S&P Global BMI Consumer Discretionary (Sector) (USD) TR	D	0.000141	0.004569	0.798816	10.25059
S&P Global BMI Consumer Staples (Sector) (USD) TR	E	0.000104	0.003034	1.125845	17.33405
S&P Global BMI Energy (Sector) (USD) NTR	F	0.000035	0.006641	1.256515	20.62600
S&P Global BMI Financials (Sector) (USD) TR	G	0.000139	0.004513	1.247199	18.96267
S&P Global BMI Gold (Sub-Industry) (USD) TR	H	0.000139	0.008295	0.101432	3.575750
S&P Global BMI Health Care (Sector) (USD) TR	I	0.000138	0.003827	0.614947	9.621013
S&P Global BMI Industrials (Sector) (USD) TR	J	0.000150	0.004118	0.917478	15.35427
S&P Global BMI Information Technology (Sector) (USD) TR	L	0.000288	0.005403	0.582073	8.870027
S&P Global BMI Materials (Sector) (USD) TR	M	0.000112	0.004441	0.736366	10.44718
S&P Global BMI Real Estate (Sector) (USD) TR	N	0.000081	0.004101	1.709118	27.94521
S&P Global BMI Utilities (Sector) (USD) TR	O	0.000125	0.003869	1.022855	22.40082
S&P Global BMI (USD) TR	P	0.000152	0.003916	1.193422	17.41700

4. Results

We applied the Bandt and Pompe method [31] to compute PE and FIM, following the approach as in [32]. Using these measures, we constructed the SFCP plane to assess the level of disorder and randomness in the daily closing prices of 15 S&P Global BMI sectoral indices. The SFCP enables visualization of market efficiency by positioning each index based on its PE and FIM values. Additionally, we analyzed the behavior dynamics of shuffled time series as a benchmark, employing $1000 \times N$ transpositions for each series.

Figure 3 and Table 2 together provide a detailed analysis of the informational efficiency of the 15 S&P Global BMI indices, leveraging their representation on the SFCP and associated metrics. Figure 3 illustrates the SFCP, representing the interplay between randomness and predictability for the 15 S&P Global BMI sectoral indices. Table 1 supplements this with numerical data for the entropy, FIM, informational efficiency (IE), distance to the ideal position (Dist. (1,0)), and a ranking based on proximity to the random ideal position (PE = 1, FIM = 0). The ideal position represents a theoretical benchmark representing maximum entropy, minimal predictability, and complete informational efficiency. Entropy reflects the degree of randomness or disorder in the time series, with higher values indicating greater randomness. The FIM quantifies predictability, where lower values suggest greater disorder. IE combines these measures to assess the overall efficiency of the index, with higher values reflecting closer alignment to efficient market behavior. Each point in Figure 3 corresponds to a specific index, with PE on the horizontal axis and FIM on the vertical axis. The red dots indicate the random ideal position. The indices' relative positions to the random ideal point reveal differences in market behavior. Smaller distances from the ideal position signify higher efficiency, as seen for S&P Global BMI Utilities (O) and Consumer Staples (E). These indices exhibit high entropy and low FIM, placing them close to the random ideal position in the SFCP and suggesting they behave in a manner similar to efficient markets with minimal exploitable patterns. In contrast, indices such as S&P Emerging BMI (B) and S&P Global BMI Financials (G) are farther from the ideal position. These indices have lower entropy and higher FIM, indicating less disorder and greater predictability, characteristics typically associated with lower market efficiency. The ranking column in Table 1 provides a summary of efficiency levels across the indices and aligns closely with the distances, emphasizing the efficiency hierarchy across sectors. For instance, Utilities (O) is ranked first, followed by Consumer Staples (E), while Emerging BMI (B) and Financials (G) are ranked lowest in terms of efficiency.

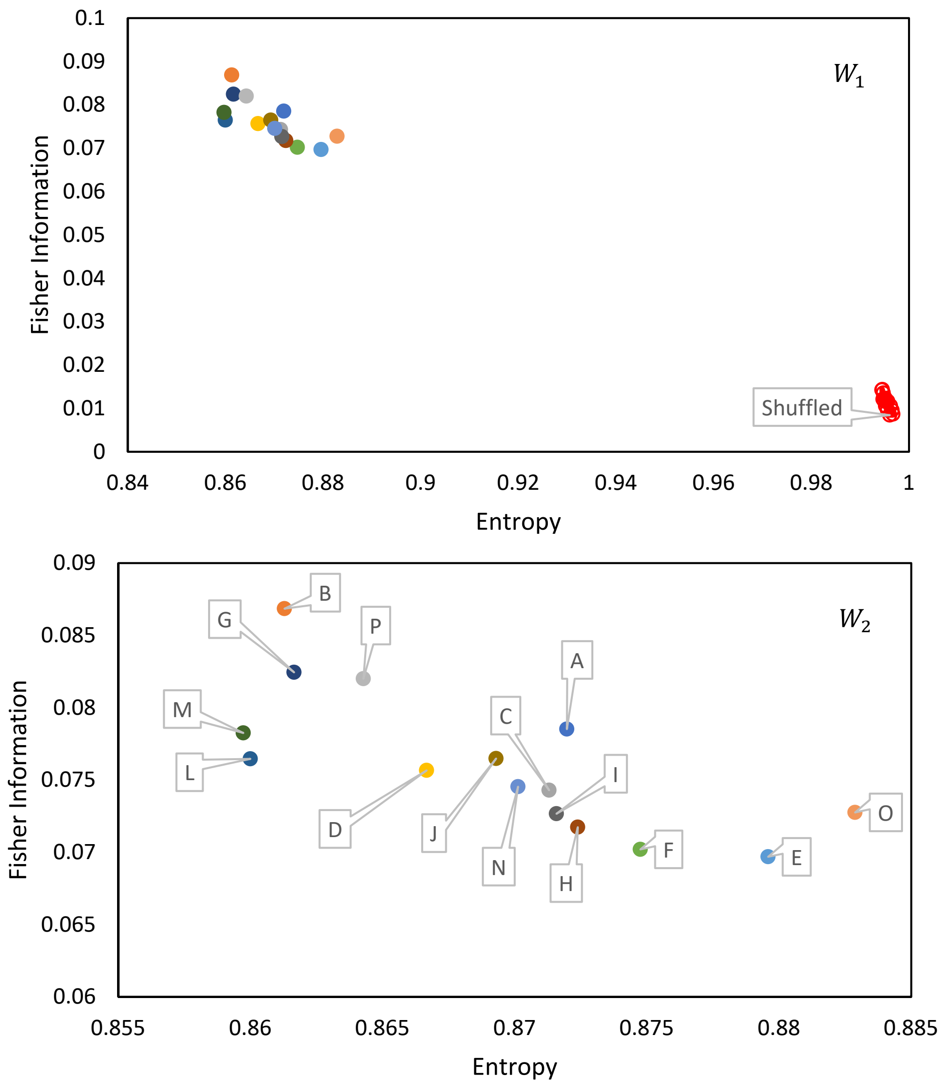


Figure 3. Shannon–Fisher causality plane for S&P Global BMI sectoral indices.

Table 2. Informational efficiency metrics for S&P Global BMI indices.

Full Name	Entropy	FIM	IE	Dist. (1,0)	Ranking
S&P Global BMI Utilities (Sector) (USD) TR	0.883	0.073	0.810	0.138	1
S&P Global BMI Consumer Staples (Sector) (USD) TR	0.880	0.070	0.810	0.139	2
S&P Global BMI Energy (Sector) (USD) NTR	0.875	0.070	0.805	0.144	3
S&P Global BMI Gold (Sub-Industry) (USD) TR	0.872	0.072	0.801	0.146	4
S&P Global BMI Health Care (Sector) (USD) TR	0.872	0.073	0.799	0.148	5
S&P Global BMI Communication Services (Sector) (USD) TR	0.871	0.074	0.797	0.149	6
S&P Global BMI Real Estate (Sector) (USD) TR	0.870	0.075	0.796	0.150	7
S&P Developed BMI (USD) TR	0.872	0.079	0.793	0.150	8
S&P Global BMI Industrials (Sector) (USD) TR	0.869	0.076	0.793	0.151	9
S&P Global BMI Consumer Discretionary (Sector) (USD) TR	0.867	0.076	0.791	0.153	10
S&P Global BMI Information Technology (Sector) (USD) TR	0.860	0.076	0.784	0.160	11
S&P Global BMI (USD) TR	0.864	0.082	0.782	0.159	12
S&P Global BMI Materials (Sector) (USD) TR	0.860	0.078	0.781	0.161	13
S&P Global BMI Financials (Sector) (USD) TR	0.862	0.082	0.779	0.161	14
S&P Emerging BMI (USD) TR	0.861	0.087	0.774	0.164	15

Now, we further elaborate on the IE index. The degree of IE is computed as the difference between PE and FIM, IE = PE − FIM, as in [22]. This index combines the randomness and informational structure of the system, offering insight into its efficiency. Specifically, this index is interpolated within the range of [−1,1], where IE = 1 represents entirely random and efficient behavior (i.e., PE = 1, FIM = 0), and IE = −1 corresponds to completely regular and inefficient dynamics (PE = 0, FIM = 1). Intermediate values capture varying degrees of efficiency. The analysis examines the dynamics of IE for these assets across three distinct periods: before COVID-19, during COVID-19, and during the Russia–Ukraine war.

Figure 4 illustrates the evolution of the informational efficiency index for each of these periods, providing insights into how these significant global events influenced market efficiency. Additionally, Table 3 presents the percentage differences between the selected periods, calculated using sliding windows of size w = 120 days (6 months) and a sliding step of Δ = 21 days (1 month), offering a quantitative summary of the percentage changes in IE between these periods.

Table 3. Percentage values of differences between the selected periods.

Full Name	Before COVID-19	During COVID-19	Russia–Ukraine War
S&P Developed BMI (USD) TR	-	−0.84%	3.18%
S&P Emerging BMI (USD) TR	-	−3.71%	10.40%
S&P Global BMI (USD) TR	-	−2.47%	2.69%
S&P Global BMI Communication Services (Sector) (USD) TR	-	−2.84%	2.60%
S&P Global BMI Consumer Discretionary (Sector) (USD) TR	-	−7.99%	13.35%
S&P Global BMI Consumer Staples (Sector) (USD) TR	-	−4.26%	2.32%
S&P Global BMI Energy (Sector) (USD) NTR	-	1.43%	−6.39%
S&P Global BMI Financials (Sector) (USD) TR	-	3.73%	−6.88%
S&P Global BMI Gold (Sub-Industry) (USD) TR	-	−2.59%	−2.56%
S&P Global BMI Health Care (Sector) (USD) TR	-	1.48%	−1.21%
S&P Global BMI Industrials (Sector) (USD) TR	-	1.94%	−4.82%
S&P Global BMI Information Technology (Sector) (USD) TR	-	0.41%	1.46%
S&P Global BMI Materials (Sector) (USD) TR	-	2.35%	−1.46%
S&P Global BMI Real Estate (Sector) (USD) TR	-	0.33%	1.63%
S&P Global BMI Utilities (Sector) (USD) TR	-	−2.73%	0.36%

During the COVID-19 pandemic, a marked decline in efficiency was observed for most indices. Sectors such as Consumer Discretionary and Consumer Staples experienced the largest reductions in IE, with percentage declines of −7.99% and −4.26%, respectively. These results highlight the heightened disorder and reduced predictability in these sectors due to the pandemic's unprecedented impact on consumer behavior and supply chains. However, Financials (+3.73%) and Materials (+2.35%) exhibited resilience, reflecting their critical importance during COVID-19. These results highlight the capacity of these sectors to maintain or enhance functionality during periods of significant uncertainty. The Russia–Ukraine War introduced further variability in informational efficiency. Sectors such as Utilities (+0.36%) and Communication Services (+2.60%) demonstrated a recovery in efficiency levels, indicating their ability to adapt to the geopolitical turmoil. In contrast, sectors like Energy (6.39%) and Financials (−6.88%) faced continued inefficiencies, likely driven by increased market volatility and economic uncertainty associated with the conflict. The temporal evolution of IE highlights the differing capacities of sectors to absorb and adapt to systemic shocks. Before the COVID-19 pandemic, the indices generally maintained stable and relatively higher levels of informational efficiency, reflecting a less volatile market environment. The transition to the pandemic period disrupted this stability, with widespread declines in efficiency, particularly in sectors heavily affected by the global health crisis. The subsequent Russia–Ukraine conflict further intensified these dynamics for some sectors while providing opportunities for others to regain stability.

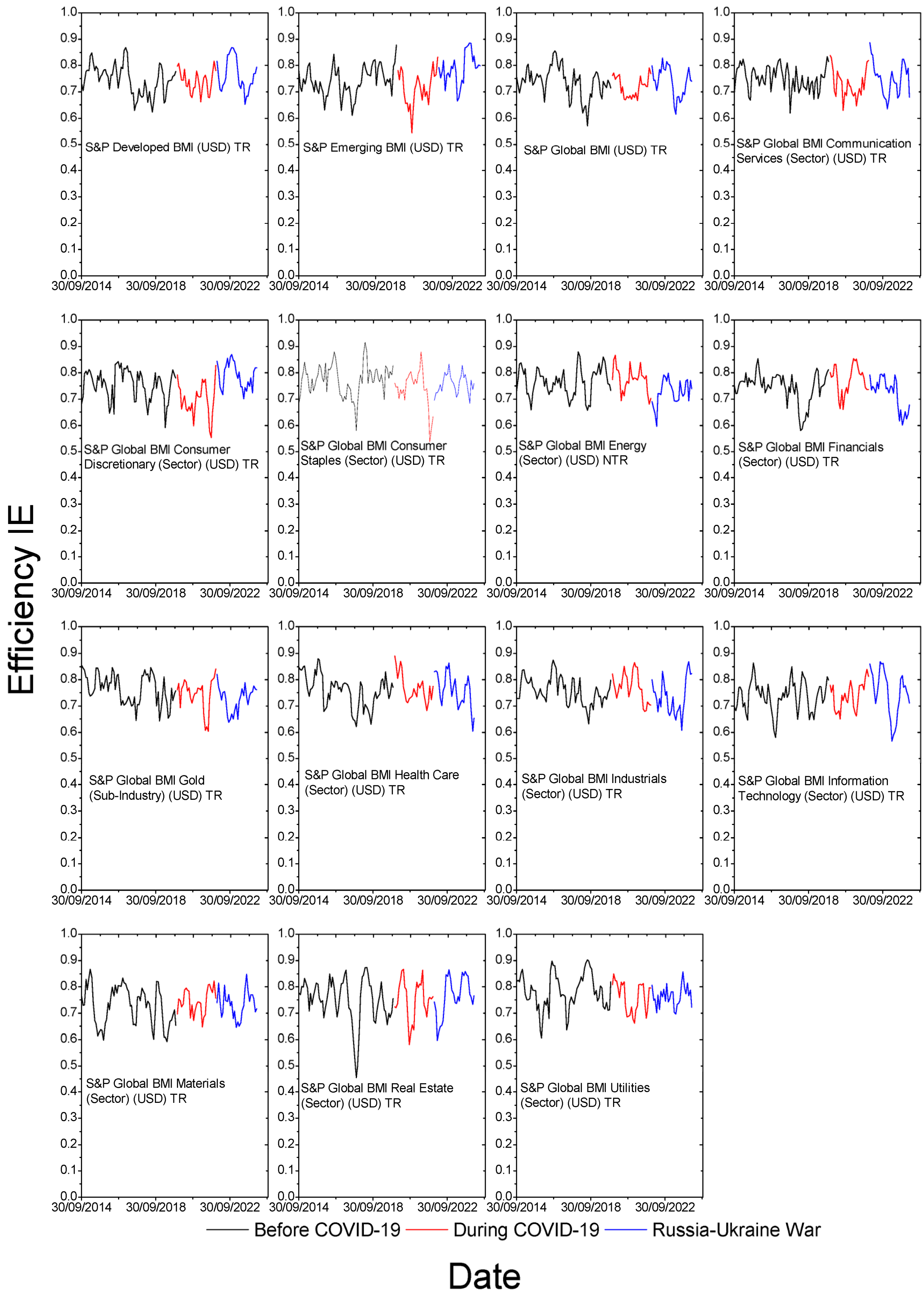


Figure 4. Temporal evolution of the information efficiency index.

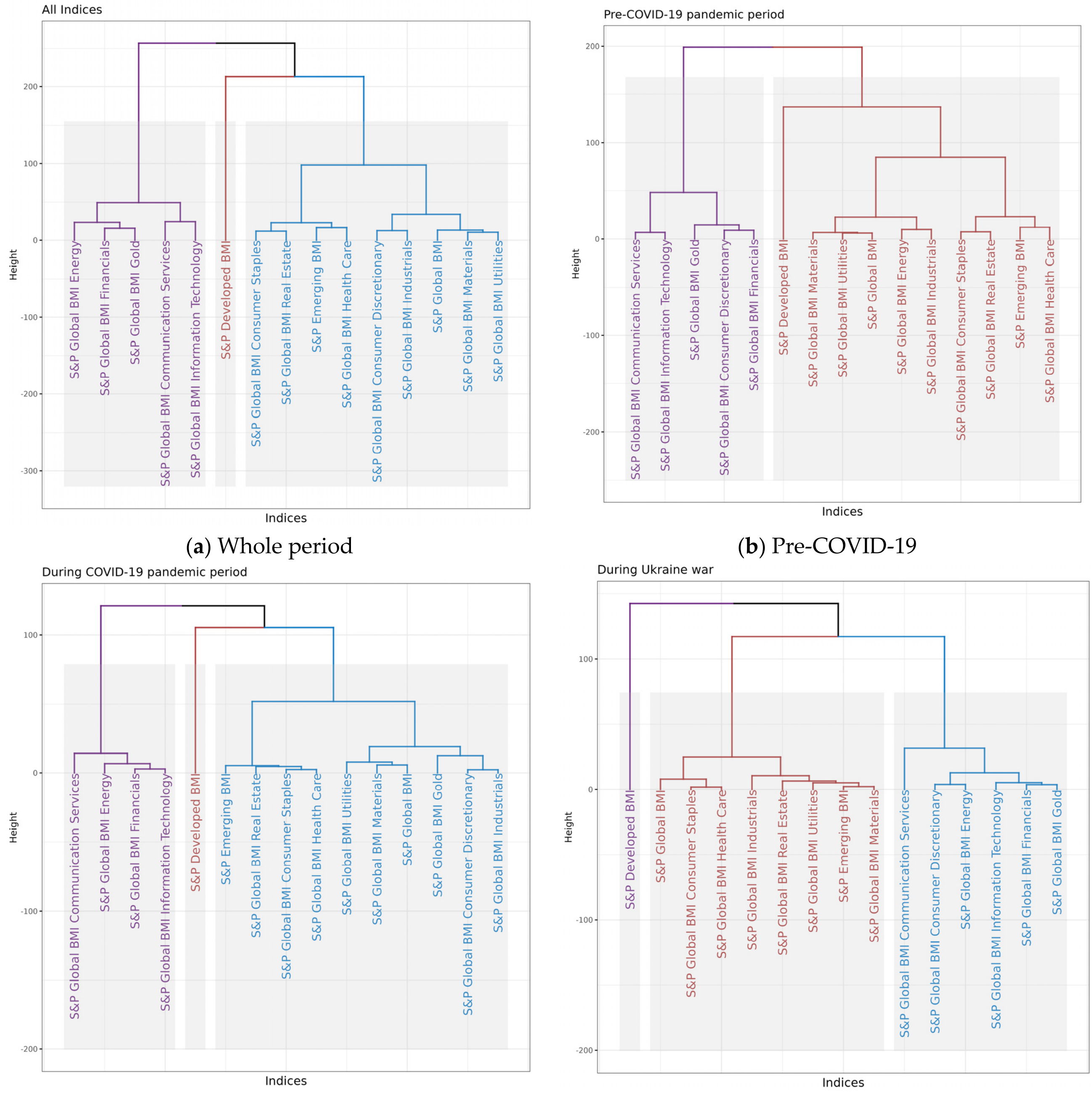
The temporal evolution further reveals sophisticated sector-specific adaptation mechanisms. Essential services sectors maintained relatively stable efficiency metrics throughout the analytical period, suggesting robust information processing capabilities even under stress conditions. Conversely, cyclical sectors demonstrated sensitivity to the transition from pandemic-induced market stress to geopolitical tensions. This sectoral divergence in efficiency dynamics carries important implications for portfolio management and risk assessment strategies during periods of compound systemic stress. Particularly noteworthy is the temporal persistence of efficiency patterns within defensive sectors. Despite initial pandemic-related disruptions, Utilities and Consumer Staples demonstrated superior efficiency recovery characteristics, maintaining their positions at the upper end of the efficiency spectrum. This resilience suggests inherent structural advantages in information processing capabilities within these sectors, potentially related to their fundamental role in economic stability and relatively predictable demand patterns. The observed temporal progression emphasizes that sectoral responses to systemic shocks are neither uniform nor linear, but rather exhibit complex patterns of deterioration, adaptation, and recovery that vary significantly across market segments and temporal phases. This heterogeneity in efficiency dynamics underscores the importance of sector-specific approaches to risk management and investment strategies during periods of market stress.

The clustering analysis examines the relationships between the S&P BMI indices under varying economic and geopolitical conditions across three periods, pre-COVID-19, during COVID-19, and the Russia–Ukraine conflict, but also across the overall considered time period.

The hierarchical clustering analysis (Figure 5) reveals two primary clusters in the pre-COVID-19 period. The first, comprising Communication Services, Information Technology, Gold, Consumer Discretionary, and Financials, reflects speculative and growth-driven dynamics. The second cluster includes all the remaining indices highlighting stable, defensive sectors. During COVID-19, Developed BMI formed a distinct cluster, indicating its unique market behavior, while a cohesive purple cluster emerged, comprising Communication Services, Energy, Financials, and Information Technology. The remaining indices are grouped into a blue cluster, emphasizing essential and resilient sectors responding to pandemic-driven volatility. The Ukraine war period saw Developed BMI remain distinct, while Consumer Staples, Health Care, Industrials, Real Estate, Utilities, Emerging BMI, and Materials formed a cohesive red cluster, reflecting stronger interconnections. Communication Services, Consumer Discretionary, Energy, Information Technology, Financials, and Gold are aligned in a blue cluster, demonstrating their behavior under war-induced conditions. Over the entire period, Developed BMI consistently stood apart, while persistent interdependencies emerged in a purple cluster (Energy, Financials, Gold, Communication Services, and Information Technology) and a blue cluster (Consumer Staples, Real Estate, Emerging BMI, Health Care, Consumer Discretionary, Industrials, Materials, and Utilities).

Similarly, the k-means clustering analysis (Figure 6) highlights significant shifts in grouping patterns across the same periods. Pre-COVID-19, two clusters were evident: one comprising Developed, Emerging, Health Care, Real Estate, and Consumer Staples, characterized by stability and alignment with essential needs and established markets, and another driven by speculative dynamics, including all the remaining indices. During COVID-19, clustering revealed three distinct groups. Information Technology, Communication Services, Gold, Financials, and Energy formed one cluster, reflecting their sensitivity to pandemic trends such as technology reliance and gold's safe-haven role. Essential and traditional sectors, including remaining indices, formed the second cluster. Developed BMI emerged as a distinct third cluster, highlighting its unique behavior during the pandemic. The Ukraine war period introduced further shifts, while keeping Developed BMI in a distinct cluster. Consumer Discretionary joined the cluster containing Information Technology, Gold, Energy, Financials, and Communication Services, reflecting strengthened interconnections. The remaining indices consolidated into a distinct cluster, emphasizing resilience. Over the entire period, Developed BMI remained separate, while Information

Technology aligned with Consumer Discretionary, Financials, Gold, Communication Services, and Energy. A third cluster of Emerging BMI, Real Estate, Health Care, Consumer Staples, Industrials, Materials, and Utilities reflected their shared stability and resilience to market shocks.

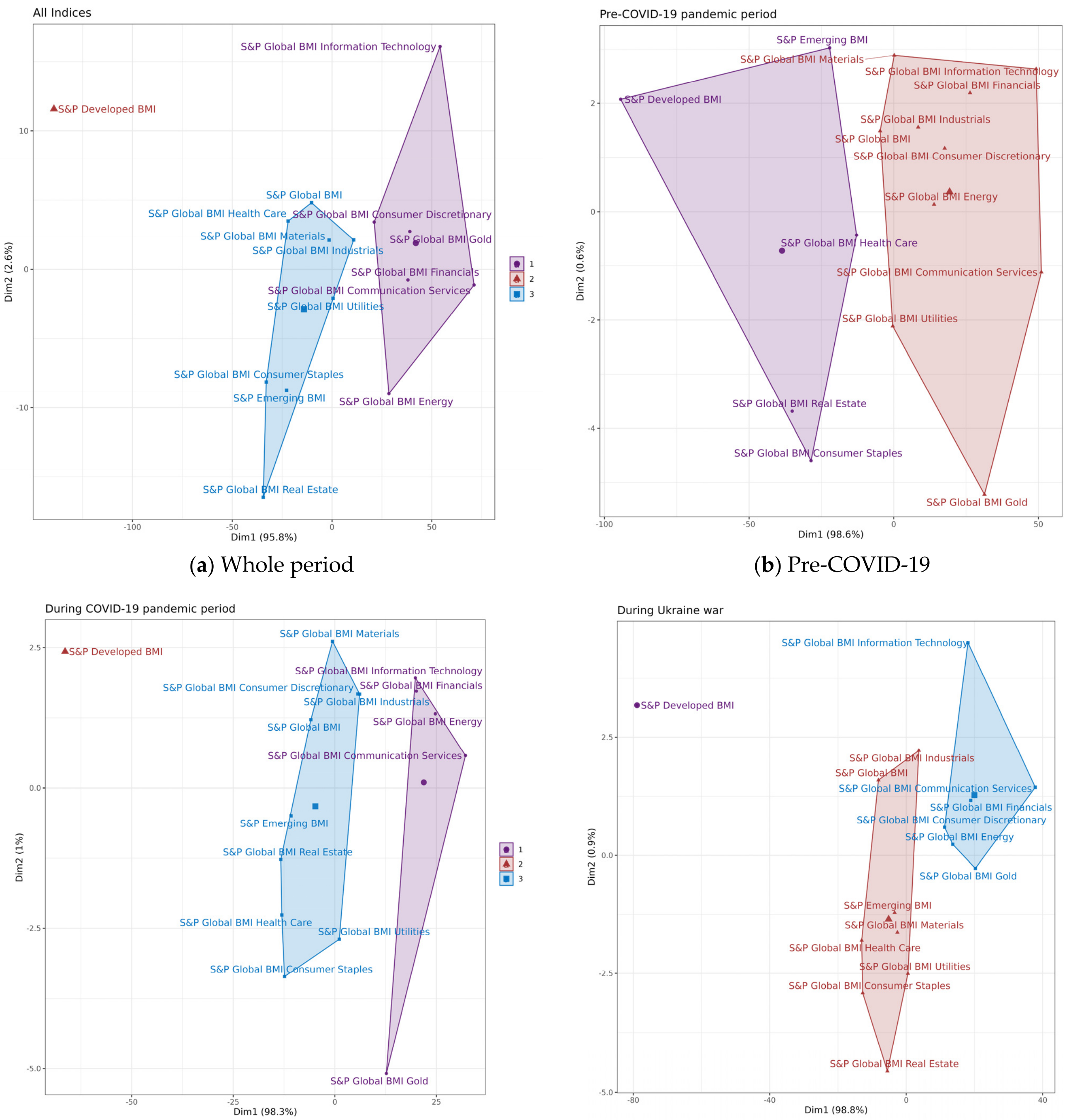


(**a**) Whole period (**b**) Pre-COVID-19

(**c**) During COVID-19 (**d**) During Russia-Ukraine war

Figure 5. Hierarchical clustering.

The stability of the clustering solution was carefully evaluated using 100 bootstrap iterations. In each iteration, the clustering algorithm was reapplied to a resampled dataset, and the resulting cluster labels were compared to those from the original clustering solution using the Adjusted Rand Index (ARI). The ARI measures the agreement between two partitions, ranging from −1 (indicating worse-than-random agreement) to 1 (indicating perfect agreement). In this context, ARI values above 0.6 are considered indicative of robust and stable clusters, reflecting a high level of consistency in the clustering structure despite data variability. For details, see Table 4.



(**a**) Whole period

(**b**) Pre-COVID-19

(**c**) During COVID-19

(**d**) During Russia-Ukraine war

Figure 6. K-means.

Table 4. Bootstrap and ARI test.

Clusters	Pre-COVID-19	During COVID-19	During Russia–Ukraine War	All Periods
1	0.684	0.604	0.7	0.684
2	0.804	0.805	0.933	0.852
3		0.725	0.935	0.859

Finally, the MFDCCA was employed to investigate the cross-correlations between the Geopolitical Risk index and various S&P BMI sectoral indices during the considered three periods and the entire analysis period. MFDCCA was applied to the logarithmic returns, calculated as $r_t = logP_t - logP_{t-1}$, with P_t representing the closing price index at time t. This transformation aligns with standard financial practice and ensures proper analysis of relative price changes across different market conditions. We have calculated and applied the coefficient of determination (R^2) to all MFDCCA graphs involving method-generated points in the dataset.

The log–log plots of Fq(s) versus s clearly demonstrated power-law scaling across all periods, confirming the presence of long-range cross-correlations (Figure 7).

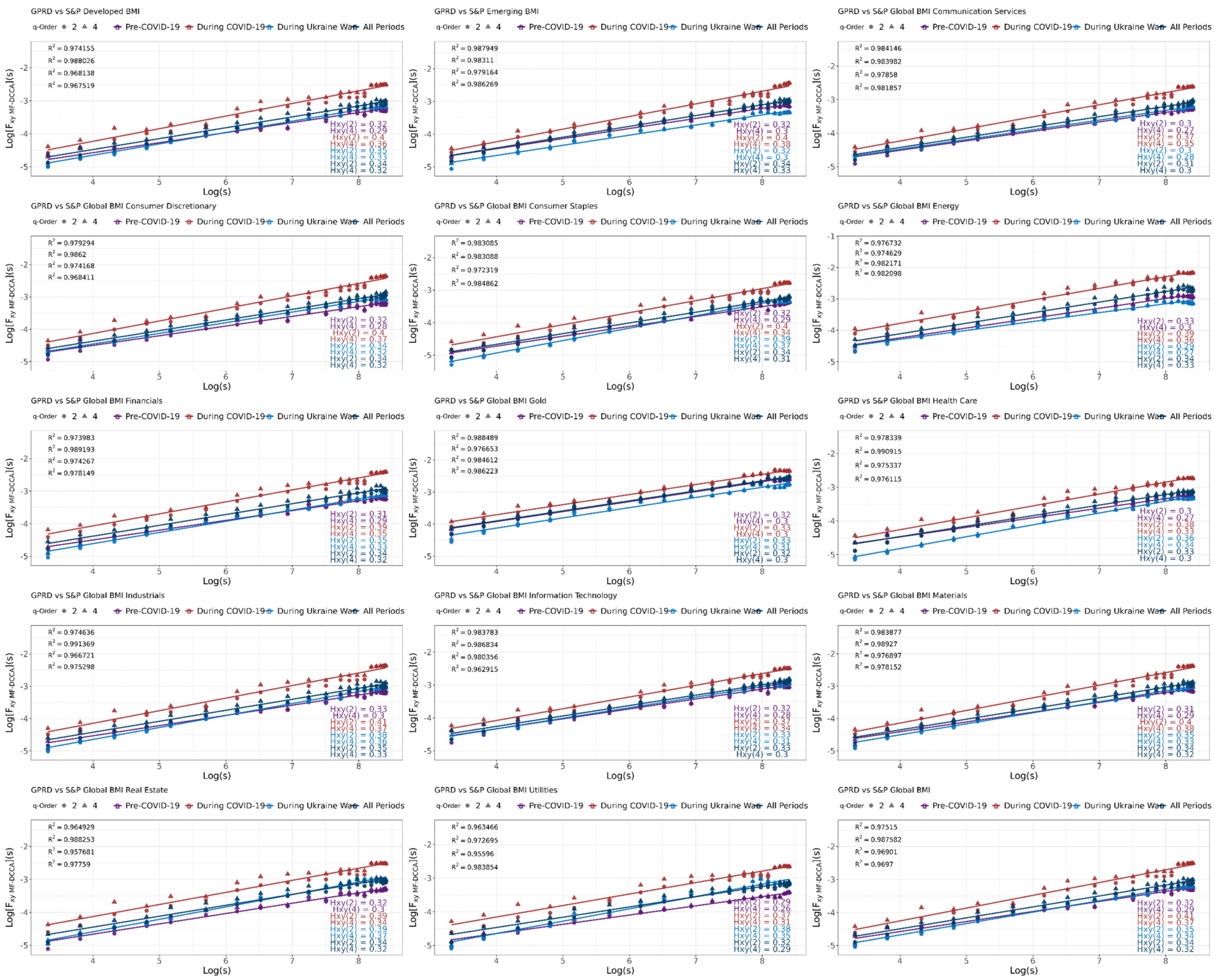


Figure 7. Fluctuation functions.

Additionally, we calculate α with q from the range −10 to 10 to obtain the multifractality quantitatively for each of the S&P BMI sectoral indices and the GPR index (Figure 8).

The cross-correlation generalized Hurst exponents $h_{xy}(q)$ for the whole considered period presented in Table 5 provide insight into the multifractal interactions between the Geopolitical Risk (GPR) index and 15 S&P Global BMI indices. For most indices, $h_{xy}(q)$ declines as q increases, demonstrating stronger multifractal correlations for larger fluctuations (negative q) compared to smaller fluctuations (positive q). This behavior is consistent with the presence of multifractality in the interactions between the GPR and the indices. Among the indices, Consumer Staples $\left(h_{xy}(q = -10) = 0.457\right)$ and Industrials

$(h_{xy}(q = -10) = 0.452)$ exhibit the highest correlations for large fluctuations, indicating a stronger response to geopolitical risk under extreme conditions. In contrast, indices such as Communication Services $\left(h_{xy}(q = -10) = 0.390\right)$ and Materials $\left(h_{xy}(q = -10) = 0.396\right)$ display lower correlations, suggesting less sensitivity to GPR in the context of large-scale fluctuations. These findings highlight the varying degrees of resilience and vulnerability of different sectors to geopolitical risk across multiple scales of market dynamics.

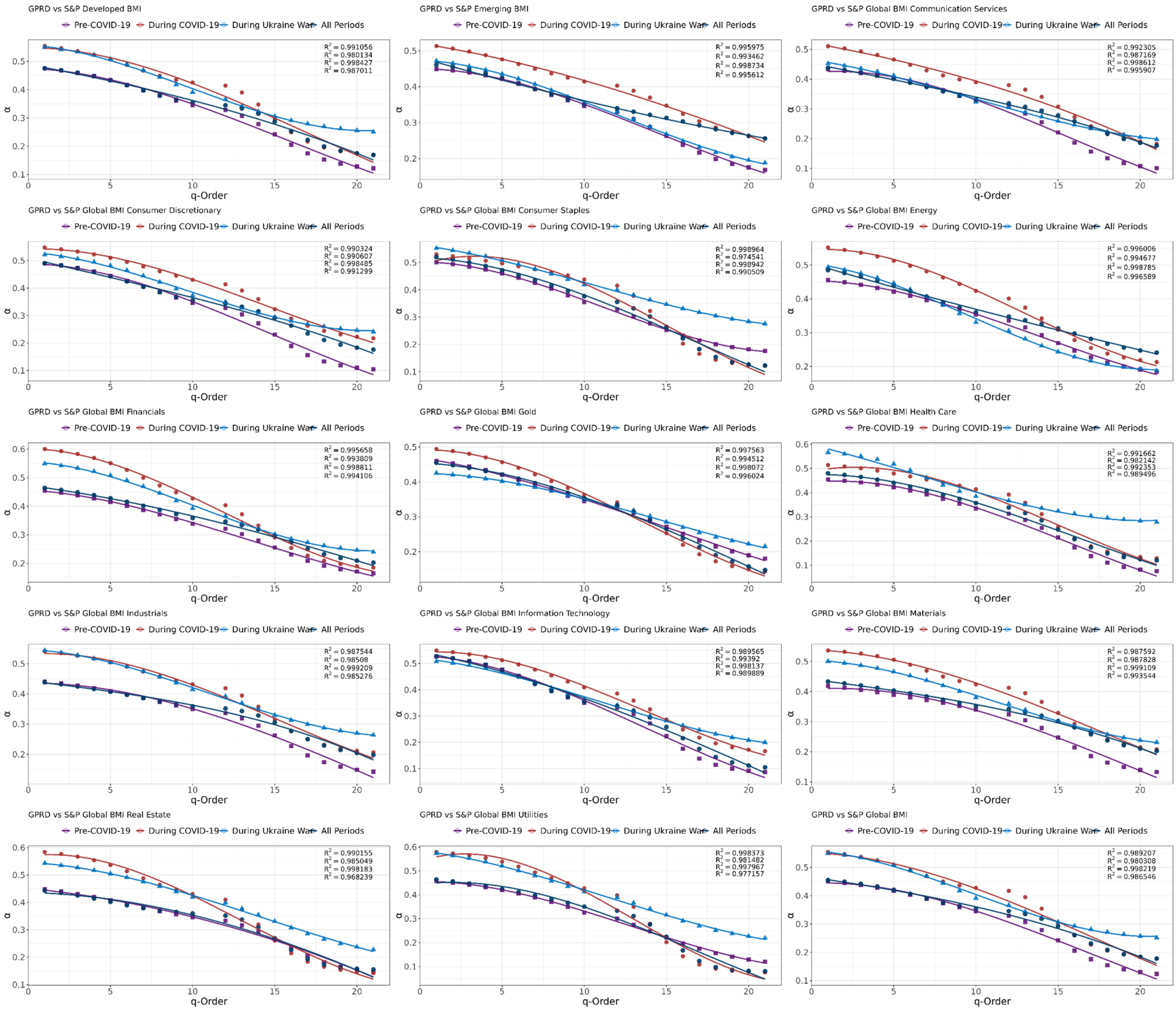


Figure 8. α as a function of q.

The multifractal spectrum analysis further supported these findings (Figure 9). The pre-COVID-19 period exhibited narrower spectra, reflecting a more uniform and stable market environment. The spectra during the COVID-19 pandemic and the Ukraine war were characterized by negative skewness, suggesting the dominance of small fluctuations in cross-correlation dynamics during these crises. Table 6 presents the multifractal statistics for the cross-correlation between the GPR index and 15 S&P Global BMI indices. The table includes three key parameters: α_0 (representing persistence), W (width of the multifractal spectrum, indicating complexity), and r (relative dominance of large fluctuations in multifractality). The α_0 values across the indices are consistently below 0.5, indicating antipersistence in the cross-correlation between the GPR index and the S&P indices. This suggests that, on average, the correlations do not exhibit long-term persistence but instead reflect alternating dynamics over time. Consumer Staples ($\alpha_0 = 0.366$) and Energy ($\alpha_0 = 0.356$) exhibit relatively higher persistence compared to other indices, indicating their more stable relationships with geopolitical risks. The W values, reflecting the width of the multifractal spectrum, capture the complexity of the cross-correlation. Higher W values indicate greater heterogeneity and a wider range of fluctuations. Information Technology ($W = 0.423$) and Consumer Staples ($W = 0.397$) demonstrate the highest complexity, suggesting that these indices experience diverse and pronounced reactions to geopolitical risk. Conversely, Communication Services ($W = 0.205$) and Energy ($W = 0.244$) display the lowest complexity, indicating less diverse responses to geopolitical fluctuations. The r parameter highlights the influence of large fluctuations on multifractality, with higher values suggesting dominance by significant changes in the series. Communication Services ($r = 1.217$) and Energy ($r = 1.115$) show the strongest influence of large fluctuations, suggesting that these sectors are particularly sensitive to major geopolitical events. On the other hand, Industrials ($r = 0.537$) and Gold ($r = 0.567$) exhibit lower r values, reflecting relatively less sensitivity to extreme events. In summary, the table underscores the diverse responses of the S&P BMI indices to geopolitical risk. Consumer Staples and Information Technology stand out for their higher complexity, while Energy and Communication Services are more reactive to large fluctuations.

Table 5. Cross-correlation generalized Hurst exponents to order q values.

GRP vs.	−10	−8	−6	−4	−2	0	2	4	6	8	10
S&P Developed BMI	0.420	0.407	0.391	0.374	0.360	0.349	0.339	0.320	0.293	0.268	0.249
S&P Emerging BMI	0.412	0.401	0.387	0.372	0.358	0.345	0.335	0.326	0.317	0.307	0.297
S&P Global BMI Communication Services	0.390	0.379	0.366	0.352	0.338	0.325	0.313	0.299	0.282	0.263	0.247
S&P Global BMI Consumer Discretionary	0.429	0.414	0.397	0.379	0.362	0.350	0.338	0.321	0.297	0.273	0.255
S&P Global BMI Consumer Staples	0.457	0.442	0.425	0.406	0.386	0.366	0.344	0.314	0.276	0.244	0.220
S&P Global BMI Energy	0.427	0.414	0.399	0.383	0.367	0.354	0.342	0.331	0.317	0.303	0.291
S&P Global BMI Financials	0.417	0.406	0.394	0.380	0.366	0.352	0.340	0.324	0.304	0.284	0.269
S&P Global BMI Information Technology	0.411	0.400	0.388	0.375	0.360	0.344	0.324	0.301	0.276	0.252	0.232
S&P Global BMI Gold	0.427	0.414	0.399	0.382	0.365	0.348	0.327	0.297	0.261	0.231	0.210
S&P Global BMI Health Care	0.400	0.392	0.382	0.372	0.362	0.355	0.347	0.332	0.309	0.288	0.270
S&P Global BMI Industrials	0.452	0.434	0.411	0.386	0.363	0.346	0.329	0.303	0.267	0.234	0.208
S&P Global BMI Materials	0.396	0.388	0.378	0.368	0.357	0.347	0.337	0.324	0.306	0.287	0.271
S&P Global BMI Real Estate	0.399	0.388	0.378	0.369	0.362	0.356	0.344	0.317	0.281	0.253	0.234
S&P Global BMI Utilities	0.413	0.402	0.389	0.374	0.359	0.343	0.322	0.286	0.239	0.202	0.178
S&P Global BMI	0.409	0.399	0.386	0.371	0.359	0.350	0.341	0.323	0.297	0.273	0.255

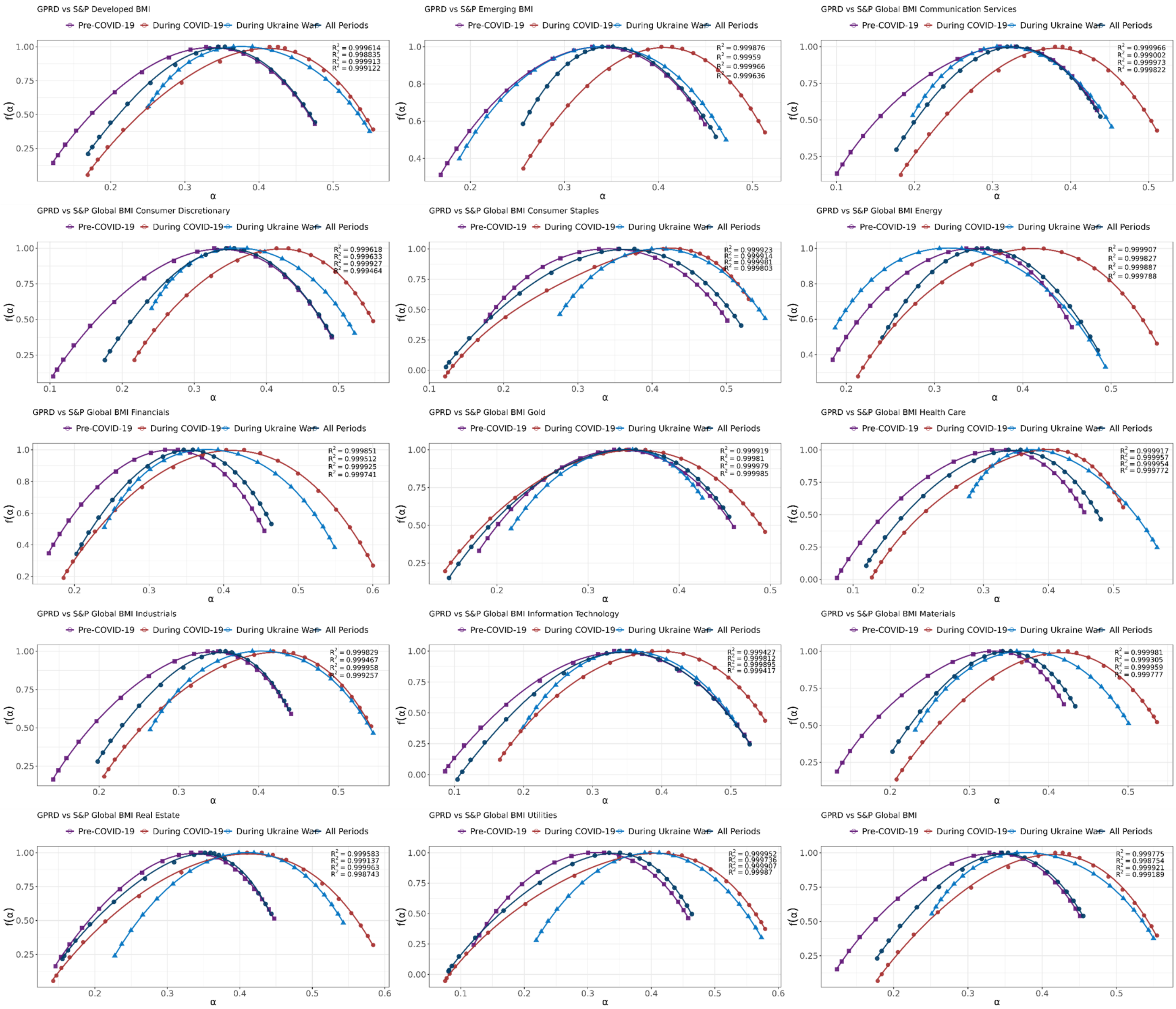


Figure 9. Multifractal spectrum.

Table 6. Multifractal statistics.

GPR vs.	α_0	W	r
S&P Developed BMI	0.348	0.306	0.712
S&P Emerging BMI	0.348	0.205	1.217
S&P Global BMI Communication Services	0.327	0.261	0.744
S&P Global BMI Consumer Discretionary	0.350	0.314	0.804
S&P Global BMI Consumer Staples	0.366	0.397	0.635
S&P Global BMI Energy	0.356	0.244	1.115
S&P Global BMI Financials	0.354	0.261	0.732
S&P Global BMI Gold	0.344	0.307	0.567
S&P Global BMI Health Care	0.347	0.360	0.589
S&P Global BMI Industrials	0.354	0.240	0.537

Table 6. *Cont.*

GPR vs.	α_0	W	r
S&P Global BMI Information Technology	0.346	0.423	0.752
S&P Global BMI Materials	0.348	0.230	0.593
S&P Global BMI Real Estate	0.349	0.288	0.488
S&P Global BMI Utilities	0.341	0.383	0.470
S&P Global BMI	0.349	0.277	0.620

5. Discussion

The discussion on the market efficiency remains a central topic in economic and financial theory, with profound implications for investors, regulators, and the economy as a whole. An efficient market, as defined by Fama [33] in his Efficient Market Hypothesis (EMH), is one in which asset prices fully reflect all available information. In this context, market efficiency can be seen as a mechanism for wealth preservation, as it reduces uncertainty and excessive volatility, enabling investors to make more informed and secure decisions. Moreover, efficient markets tend to be more resilient to external shocks, such as inflation, since prices adjust rapidly to new information, preserving the real value of assets over time [34]. On the other hand, inefficient markets, while potentially offering opportunities for opportunistic investors, such as traders seeking to exploit information asymmetries or price discrepancies, also pose significant risks. Inefficiency can lead to uncontrolled volatility, increasing the likelihood of substantial capital losses for less informed investors or those with limited risk management capabilities. This dynamic can create an environment where some participants benefit at the expense of others, leading to imbalances that may deter more conservative or institutional investors concerned with risk ratings and portfolio stability.

This study does not aim to discuss or challenge the EMH, which has been extensively explored and validated in various contexts [33,35]. Instead, it seeks to open space for reflections on the nuances and complexities of markets, recognizing that efficiency is not a binary state but a spectrum that varies depending on the context, asset type, and macroeconomic conditions. In this sense, this study leaves important questions open for future research, such as identifying optimal levels of efficiency that balance return opportunities with market stability, or how different types of investors can adapt to environments with varying degrees of efficiency.

There are studies challenging the EHM, showing that market efficiency varies, especially during crises when liquidity issues, volatility, and behavioral biases affect prices [34,35]. These disruptions often lead to inefficiencies, influenced by structural factors that shape how markets absorb shocks. While market efficiency varies across countries, institutional and economic sectoral differences [36] also emerge, with some industries proving more resilient than others [11]. Empirical research supports that Utilities and Consumer Staples tend to maintain efficiency due to their inelastic demand, as these sectors provide essential goods and services that remain stable even in volatile markets [1,8]. In contrast, the Financials sector exhibited significant inefficiencies during crises, as evidenced by increased multifractality and stronger return correlations during the COVID-19 pandemic compared to the GFC, indicating reduced market efficiency [37].

While prior studies primarily focus on single-crisis market dynamics, they often analyze regional markets or broader economic trends rather than global efficiency across multiple crises. Most works examine either COVID-19's impact on market behavior [13,38–40] or the financial implications of geopolitical risks such as the Russia–Ukraine conflict [15,16,41,42]. This study builds on these findings by comparing sectoral efficiency during both crises, revealing how industry resilience varies depending on crisis type and duration. The results confirm Utilities and Consumer Staples as efficiency-preserving sectors, while also highlighting differences in cyclical sector recoveries post-COVID-19 versus during ongoing geopolitical uncertainty.

Sectoral efficiency has direct investment strategy implications, particularly in sectoral rotation models, where investors adjust portfolios based on relative market efficiency and risk levels [43–45]. In times of crisis, investors may favor defensive sectors such as Consumer Staples and Utilities, as they retain efficiency and exhibit lower volatility, making them a potential choice for investors seeking safe-haven assets [46–48]. In contrast, cyclical sectors—such as Financials [45] and Energy—often experience heightened inefficiencies, leading investors to reallocate capital toward more resilient sectors [49]. The clustering results in this study reinforce these findings, revealing persistent linkages between defensive sectors and greater dispersion in cyclical sector responses.

Measuring market efficiency requires advanced statistical tools that capture long-term dependencies and nonlinear relationships. The Hurst exponent, introduced in fractal market theory [50], has been widely applied in financial research to assess persistence in market behavior [51]. Recent work by Nedeltchev and Zaevski [52] applies fractional Brownian motion and the Hurst exponent to study volatility behavior during crises, including COVID-19, highlighting how market turbulence alters volatility structure. However, the Hurst approach does not fully capture cross-sector dependencies or nonlinear relationships in crisis periods. This study extends prior research by integrating MFDCCA, which allows for a more detailed assessment of efficiency shifts across multiple crises [30]. Unlike traditional methods, MFDCCA identifies hidden dependencies between sectors, providing a broader perspective on market efficiency dynamics under stress.

Beyond efficiency analysis, financial risk measurement has also evolved, with traditional Value at Risk (VaR) models criticized for underestimating tail risks during extreme market events [53,54]. Conditional Value at Risk (CVaR) improves downside risk estimation [55], while newer models, such as Entropic VaR (EVaR) [56] and expectile-based risk measure [57], offer more adaptive and asymmetric risk assessments. Zaevski and Nedeltchev (2023) [58] further highlight VaR's limitations, advocating for expectile-based risk measures as a more effective alternative. Alongside these advances, other methodologies, such as Spectral Risk Measures (SRMs) [59], provide an alternative framework that assigns different weights to extreme losses, improving risk sensitivity under crisis conditions. However, SRMs have faced criticism regarding their dependence on the choice of utility functions, which determine how risk aversion is incorporated into the model [60]. Despite these concerns, SRMs remain a valuable approach for capturing tail risks, especially when calibrated carefully for specific market conditions. While this study focuses on efficiency dynamics and clustering analysis, these evolving risk measures present an opportunity for future research to integrate entropy-based risk models into multifractal efficiency frameworks to better capture sectoral resilience in financial crises.

The presented analysis highlights the role of efficiency in market stability, showing that certain sectors, such as Utilities and Consumer Staples, maintain efficiency even during crises. This stability is often seen as beneficial, reducing volatility and supporting long-term investment. However, market efficiency is not always universally positive. Some inefficiencies stem from barriers that restrict participation, while others create opportunities for excess returns and encourage active investment. In contrast, highly efficient sectors provide stability but may limit profit potential. Market efficiency should be viewed in context, considering its effects on stability, liquidity, and investment incentives.

In summary, while market efficiency is generally associated with benefits such as wealth preservation and protection against factors like inflation, inefficiency may, in certain cases, create opportunities for some investors, albeit at the cost of increasing systemic risks and imbalances. This study does not propose new theses that challenge widely accepted theories but rather discusses the vast range of possibilities that markets can present, encouraging future research to explore these nuances in greater depth. This study adds to the broader discussion on market efficiency, investment strategies, and risk modeling by examining how sectoral efficiency changes across different crises. The findings reinforce existing literature on sectoral resilience, validate clustering-based efficiency analysis, and suggest that integrating entropy-based risk measures could enhance future financial stability assessments.

6. Conclusions

This study evaluates market efficiency using multifractal measures, analyzing global financial indices during significant economic events, including the COVID-19 pandemic and the Russia–Ukraine war. The findings of this research provide actionable insights for various stakeholders, including institutional investors, policymakers, financial regulators, and academics. By examining sectoral responses to these crises, this study highlights both vulnerabilities and opportunities within the global financial system due to their roles in maintaining systemic stability and influencing market behavior, which may benefit most from these results.

Institutional investors, including asset managers, pension funds, and hedge funds, and other investors with actively managed portfolios, may navigate systemic shocks more effectively by using this study. Institutional investor portfolios could prioritize defensive sectors, such as Utilities and Consumer Staples, before a crisis to establish a stable core allocation and reduce volatility. Reducing exposure to vulnerable sectors, such as Financials, and using clustering insights to identify emerging resilient groups can help ensure the portfolio remains adaptable. Post-crisis, incrementally reintroducing cyclical and growth-sensitive sectors, supported by multifractal analysis to evaluate residual risks, may optimize recovery and long-term stability.

Policymakers and financial regulators can use these findings to support key sectors based on their resilience during crises. Utilities and Consumer Staples, which ensure economic continuity, should be supported with capital access, regulatory frameworks, and operational stability during crises. Financials, which showed vulnerabilities, requires enhanced liquidity buffers and stronger risk management frameworks to withstand shocks. It is recommended that policymakers develop early-warning systems for identifying sectoral vulnerabilities and implement preemptive fiscal measures, such as targeted liquidity measures, to mitigate potential shocks before they escalate. Meanwhile, Healthcare and Communication Services, which adapted well during crises, should receive targeted support, including emergency funding, to strengthen their crisis response capabilities. Technology and energy sectors, with varying resilience, need adaptive crisis-specific policies to ensure long-term stability and preparedness for future shocks. In particular, policymakers should implement adaptive regulatory frameworks to help the Energy sector transition to sustainable practices while ensuring the Technology sector remains robust enough to handle increased demand and mitigate risks from emerging technologies like AI.

As discussed earlier, efficiency can contribute to market stability, but it is not universally beneficial. While some inefficiencies result from structural barriers, others create opportunities for investors without necessarily increasing market instability. This study reaffirms that sectoral efficiency and resilience depend on broader market conditions, reinforcing the need to assess efficiency in context rather than as an absolute measure of market quality.

To address these findings comprehensively, it is essential to consider the broader implications of overlapping crises and the systemic risks they present. The interplay of overlapping crises, such as geopolitical conflict overlapping with health- or climate-related shocks, demands integrated approaches that account for cumulative impacts on financial systems. Coordinating efforts across sectors and borders will be necessary to mitigate cascading risks and ensure resilience.

Future research should build on these findings to explore sectoral resilience across varying crises, such as climate-related events, global and localized economic shocks, health-related disruptions, conflicts and political crises. Examining regional differences could provide a more nuanced understanding of how different geographies influence sectoral interdependencies. Expanding the temporal scope to analyze both long-term structural changes and short-term shocks would further clarify the dynamics of sectoral resilience. Additionally, incorporating new indicators, such as sustainability performance metrics, economic policy uncertainty metrics, and macroeconomic variables could provide additional insights into the internal and external drivers of resilience. Finally, extending the scope to consider systemic risks, such as technological disruptions, cyberattacks, and demographic shifts, would enhance understanding of vulnerabilities and adaptive capacities across sectors. Future research could explore the underlying drivers of sectoral efficiency and resilience, as well as the potential trade-offs between efficiency, stability, and investment opportunities. This would provide a more comprehensive understanding of the complex interplay between market efficiency and sectoral dynamics during periods of crisis.

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