

Ivana Simeunović

Faculty for Banking, Insurance and
Finance, Union University, Belgrade

Ivana Domazet

Institute of Economic Sciences, Belgrade

Jovan Zubović

Institute of Economic Sciences, Belgrade

MODELLING A BMS-BASED AUTOMOBILE LIABILITY RATING SYSTEM: THE SERBIAN CASE*

Modeliranje premijskih stopa autoodgovornosti u Srbiji
primenom bonus-malus sistema

Abstract

In this paper the authors examine the viability of applying a bonus-malus system in establishing automobile liability premium rates. They have developed an optimal tariff system which reflects the equivalence principle between the amounts of the premiums and the reported losses of individual policyholders. Based on past incidents of the policyholders, a homogeneous distribution of the risk classes and an improved insurance rating have been obtained. The developed model was tested with the negative binomial distribution and the expected value principle on a sample of 98,978 policyholders in Serbia. As a result, the set tariff system has been financially balanced, i.e. a balance was struck between the cost amounts of total premiums among policyholders and the cost amount of expected future claims. It has been proven that, in the process of modelling the automobile liability rating system based on the frequency of claims, it is optimal to use sophisticated distribution models such as the negative binomial distribution.

Keywords: *bonus–malus system, negative binomial model, Bayesian inference*

Sažetak

U ovom radu autori se bave problemom primene bonus-malus sistema u procesu formiranja premijskih stopa autoodgovornosti. U tom smislu, u radu je formiran optimalni sistem tarifa predmetnog osiguranja koji odražava načelo ekvivalencije između iznosa premije i broja prijavljenih šteta individualnih osiguranika. Poznati oblik iskustvenog utvrđivanja premije, bonus-malus sistem (BMS), podrazumeva analizu prethodnog iskustva osiguranika, čime se obezbeđuje postavljanje homogenih klasa rizika i unapređuje postupak tarifiranja. Opisani tarifni sistem je kreiran primenom modela negativne binomne distribucije i principa očekivane vrednosti na uzorku od 98.978 osiguranika u Srbiji. Postavljeni tarifni sistem je finansijski izbalansiran, odnosno uspostavljena je ravnoteža između iznosa ukupnih premija i iznosa budućih očekivanih šteta. Time je dokazano da je u procesu modeliranja optimalnih tarifa autoodgovornosti, koji se bazira na frekvenciji odštetnih zahteva osiguranika, veoma korisno oslanjati se na izvedene distribucije kao što je negativna binomna.

Ključne reči: *optimalni bonus-malus sistem, model negativne binomne distribucije, Bajesovski pristup*

* This paper is a part of research projects numbers 47009 (*European integrations and social and economic changes in Serbian economy on the way to the EU*) and 179015 (*Challenges and prospects of structural changes in Serbia: Strategic directions for economic development and harmonization with EU requirements*), financed by the Ministry of Science and Technological Development of The Republic of Serbia.

Introduction

Automobile liability insurance is a form of insurance that protects third party claimants from damages inflicted by the use of a motor vehicle. In most countries, this type of insurance is mandatory, and it is dominant in the total of non-life insurance portfolio. Considering the competitive environment in which they operate, auto liability insurers are facing a problem to maintain cross-subsidies among different risk categories [4]. Therefore, it is necessary to establish a system of tariffs that will reflect the risk of an individual policyholder. The issue of developing an appropriate premium system is very important when considering the overall business of the automobile liability insurers.

The aim of this paper is to develop an optimal automobile liability tariff system, which includes the establishment of equivalence between the number of reported claims of a policyholder and the amount of his/her premiums. In addition to contributing to the competitiveness of the automobile liability insurance market, the implementation of this so-called optimal tariff system is one of the most effective methods to increase traffic safety and to promote additional punishment of careless and unscrupulous drivers. This allows the implementation of one the basic principles that should reflect the price of insurance – the principle of fairness.

The procedure used to penalise the policyholders responsible for activation of the insurance policy by increasing the basic premium, or by granting discounts to individuals who are not incurring damages, known as the bonus-malus system (BMS), is an integral part of tariff systems in practically all the insurance companies in the world. The BMS is virtually the mechanism that strikes the balance between the cost of insurance and previous conduct of the policy holder [1, pp. 11-32], [4, pp. 577-599], [10, pp. 199-212], [11, pp. 196-204].

The described tariff frame was set in the 1910s in England and the Scandinavian countries, while the first actuarial study dedicated to this tariff model emerged in the early 1960s [2, pp. 106-112], [3, pp.119-130], [7, pp. 84-95]. The most significant contribution in this field,

according to many authors, was given by Bühlmann [5] and Lemaire [15], [16], [17].

This paper is organised in the following way: the first part describes the bonus-malus system with the aim to highlight the most important features of this system of tariffs, as well as its various applications. The second part of the paper encompasses the problem of constructing the optimal BMS. In this regard, at the beginning of the paper the authors present the well-known Bayesian credibility premium model, and then analyse the problem of modelling the claim counts in automobile liability insurance. Specific attention is given to several common models that are applied in practice. The third part of this paper delivers research results achieved by applying the previously described models on the selected sample, while selecting an appropriate probability distribution to which the obtained data are adjusted. The last part of this paper introduces the most important conclusions.

Literature review on BMS

The process of establishing tariffs, as a key task of an actuary, basically involves the allocation of risks to an individual policyholder according to the possible claims that could be reported. The basic idea in the a priori rating process is included in the classification of policyholders into homogeneous risk classes according to their anticipated characteristics.

The BMS is used in virtually all the countries and is practically based on the concept of fairness. In contrast to the a priori tariff system, which penalises bad drivers, it represents a set of developed actuarial models based on a posteriori concept of evaluation of individual risk factors in automobile liability insurance. Implementation of this model introduces a balance between the probability that the insured is a serviceable and yet an unfortunate driver who has suffered damage, and the possibility that the insured is a really poor driver, which is why he/she should be penalised with higher premiums. Henriot and Rochet [13] point out the two advantages of this model: it solves the problem of the adverse selection – nothing counts but the frequency of reported claims, and the problem known as the moral hazard – the damages that occur over time

should be the main indicator for establishing tariffs in order to encourage drivers to drive more carefully.

In the case of comprehensive insurance, bonuses (discounts) are usually activated at the end of one or multi-year periods, while in the third party automobile liability insurance they are approved in advance. When determining the eligibility for a discount, the common practice is that damages caused by other persons are ignored, so the bonus is often called the discount for lack of guilt.

If Y_i denotes the random variable that represents one of the values significant for the process of quantifying the risk in automobile liability insurance for the policyholder – i (e.g. number of reported claims in a given period), it is clear that the value of this variable will be a function of both predefined values: X and Z . Therefore, Denuit and Dhaene [8, pp.13-27] define automobile liability insurance cost as the sum of purely random fluctuations of the risk and the variation of the expected claims due to the unknown characteristics – Z , which can be expressed by the following relation:

$$E[\text{Var}[Y_i|X_i]] = E[\text{Var}[Y_i|X_i, Z_i]] + E[\text{Var}[E[Y_i|X_i, Z_i]|X_i]] \quad (1)$$

When it comes to the implementation of the BMS, the basic characteristic of such tariff system is the existence of the base premium, which, together with the discount level, determines the price of the insurance that an individual will pay. In addition, the BMS of each country independently determines the discount level for new policyholders. After each year, the policyholder moves up or down the scale in accordance with the defined transition rules and the last year's performance. During the XX century, a linear BMS was applied in most European countries. Yet in 1994, the European Union adopted a decree abolishing the compliance of mandatory tariff systems. It was emphasised that this system was suppressing market competition, which was not in accordance with the principle of "freedom" in the process of establishing automobile liability insurance tariffs. However, uniform and mandatory bonus-malus tariffs are still applied in France. The legislation enforced by the government has forced the automobile liability insurers to use a uniform BMS, which included an identical application of this highly important instrument for the process of the establishing premium rates. In addition, it

is interesting to note that the system does not function as described above, since there is no reduction scale.

On the other hand, the CRM (coefficient de réduction-majoration) is used in the process of establishing the premiums. This quite simple concept implies the following: each policyholder who does not report the claim within one year will receive a 5% discount on the basic premium, while in the case of an accident for which the insured is responsible, the premium will be increased by 25%. So, in the event of a reported claim for which he/she is responsible, the base price of insurance will be multiplied by 1.25, while each year without any reported claim will imply a new premium price, reduced to 95% of the previous price. In addition to this, the maximum amount of the premiums can reach up to 350% of the base price; while the maximum discount, in other words the best price, can be reached after 13 consecutive years without claims and it amounts to 50% of the base premium [9]. The specific application of the BMS and determining the rules for teetering through the discount scales, maximum discount, malus attitude in relation to the number of reported claims, and so forth, are directly dependent on the economic development of the respective country. Naturally, the more developed countries tend to enforce a stricter BMS, whereas, in developing countries a simple BMS with just a few classes and elemental transition rules is applied in practice. Park et al. [20, p. 25] show that as insurance markets become more mature and policyholders become more sophisticated, countries are transferring to more severe BMSs.

Modelling the optimal BMS

From the policyholders' point of view, the bonus-malus system is optimal if it is financially balanced. However, Wang and Zhou [23] argue that, from the standpoint of the policyholder, it is fair if it is proportional to the individual risk.

Setting up a tariff framework based on previous conduct and the principle of fairness has been discussed in various actuarial works. Several policies, aimed to solve the aforementioned problem, have been developed. According to one developed by Lemaire [16], [17], the design of an optimal

BMS is achieved by using the negative binomial model with the expected value principle. Instead of the negative binomial distribution, for modelling the claim counts, Meng and Whitmore [18] developed a model based on the Pareto distribution. Tremblay [22, pp. 97-106] analyses the problem of establishing the optimal bonus-malus scale using the Poisson-inverse Gaussian model, while using the zero utility principle in the process of determining the premium tariffs. In contrast to the abovementioned and other approaches that have been developed entirely on the basis of information on the number of claims reported, Frangos and Vrontos [12, pp. 1-22] developed a model for the design of the optimal BMS that analyses claim frequency and severity, as well, by using Poisson-gamma as the claim frequency distribution and the exponential-inverse gamma distribution as the severity distribution. The most common problem in applying the said model arose from the policyholders whose claims were just above the limit. Another solution, with favourable empirical results, considers classification of all claims in two categories depending on the nature of the harmful effects thereof. Thus, all claims for which the consequences are solely of a material nature are classified into one group, while claims including injuries are analysed independently. Moreover, Jovković and Ljubisavljević [19] apply the variable sampling method in testing the premium income.

The structure of the process explained in the following sections is the following: first, we shall present the model of structuring an optimal BMS under the Bayesian analysis called the Bayesian credibility premium model; then we shall describe the problem of modelling the claim counts. In this sense, the χ^2 goodness-of-fit test was used to compare candidate fitting distributions. This represents the beginning of the process of establishing the net premium based on the characteristics of preselected distribution. Different models may be applied in this procedure: the expected value principle, the standard deviation principle, the variance principle, the principle of zero utility, and many more. As noted, this paper relies on the expected value principle, whose requirements and application are presented in the last stage of the process of establishing an optimal BMS.

The tariff system established in this paper is derived from the data on reported claims, and is not based on

the cost amounts of these claims. Also, the analysis does not include some very important a priori characteristics of the driver that can have a considerable influence on the origination of damages. These represent the basic limitations of the established tariff system, in addition to the fact that most of the countries use significantly less severe BMSs in practice. This particularly applies to the length of a claim-free period, which is necessary to eliminate the influence of a reported claim.

Three basic approaches used to determine the frequency distribution and the claim amount are: empirical, analytical and the moments-based methods.

In the event of existence of the corresponding databases containing a large amount of data, it is possible to use the empirical method to run a smooth and accurate assessment of the cumulative distribution function. The moments-based method comprises evaluation of the moments of distribution, usually the mean and variance, and has a greater application in modelling the claim severity. The most widely accepted in the actuarial literature and in practice is the analytical method that involves finding an appropriate analytical expression that could describe the observed data.

Since the claim frequency modelling process implements a large number of distribution functions, it is necessary to reduce their number. One of the following distributions is applied to claims in third party automobile liability insurance: Poisson, negative binomial and Poisson-inverse Gaussian. In order to determine the analytical expression of automobile liability claim frequency, it is necessary to create a model for each of the three distributions. However, it is important to address the issue of selecting the appropriate model in the selected sample. Note that, in terms of probability calculations and given that known functions depend on a finite number of parameters, each of the models is parametric.

Poisson model

Poisson distribution is among the most important distributions used to determine the probability of the number (frequency) of claims in a certain time period or area. Its use is valid for large events whose probability of occurrence is very

low, where the application of this distribution is based on the assumption of homogeneity of the population on which the respective analysis is conducted. It occupies a dominant position in modelling claims per unit of time for the insured individual. A random variable, describing the number of claims in a given time interval, could be represented as a Poisson random variable $X: Poi(\theta)$, and has a probability mass function:

$$p_k = e^{-\theta} \frac{\theta^k}{k!}, \theta > 0 \tag{2}$$

The mean and the variance of the Poisson are both equal (this feature is also known as equidispersion), where θ is equal to the average number of occurrence of an event (damage) per unit of time.

$$E(X) = Var(X) = \theta \tag{3}$$

Estimates of values for parameters in this distribution can be derived by applying the maximum likelihood method or the method of moments. Since the first and second moments in the observed distribution are equal, this model is, usually, rejected in practice. The reasons for its rejection shall be analysed on a selected sample of policyholders.

Negative binomial (Poisson-gamma) model

Let us assume that the frequency of claims for the individual automobile liability portfolio can be approximated by a Poisson distribution, in which the parameter θ of the Poisson distribution takes on different values. Thus, each policyholder is characterised by a certain value of this parameter, which means that the behaviour of each insured person is represented by a single obtained value θ of the random variable Θ .

The following equation is known as the mixed Poisson distribution:

$$p_k = \int_0^\infty e^{-\theta} \cdot \frac{\theta^k}{k!} g(\theta) d\theta, k=0,1,2,\dots \tag{4}$$

where $g(\theta)$ represents the density function of the random variable Θ .

Furthermore, let us assume that the parameter of the Poisson distribution Θ has a gamma distribution whose parameters are a and b , where $\Theta: \Gamma(a, b)$ applies:

$$g(\theta) = \frac{b^a e^{-b\theta} \theta^{a-1}}{\Gamma(a)} g(\theta) d\theta, a, b > 0 \tag{5}$$

The resulting distribution of the number of claims in the portfolio, known as the negative binomial distribution, will then have the following form:

$$p_k = \binom{k+a-1}{k} \left(\frac{b}{1+b}\right)^a \left(\frac{1}{1+b}\right)^k = \binom{k+a-1}{k} p^a (1-p)^k \tag{6}$$

whose first and second moments are equal, respectively:

$$E(X) = \frac{a}{b} \quad \text{and} \quad \sigma^2 = \frac{a}{b} \left(1 + \frac{1}{b}\right) \tag{7}$$

Estimation of the unknown parameters of the described distribution is calculated by a maximum likelihood method or the method of moments.

Poisson-inverse Gaussian model

Assuming that the claims frequency for the insured individual has a Poisson distribution, where the “behaviour” of each policyholder is determined by a single θ realisation of the random variable Θ , we shall review the case when the Poisson distribution parameter Θ adjusts to the inverse Gaussian distribution [14], [21].

Thus, the random variable is distributed according to the inverse Gaussian distribution, where $X: IG(\alpha, \beta)$ applies, if its density function can be represented by the following function:

$$f(x) = \frac{\alpha}{\sqrt{2\pi\beta x^3}} e^{\left(-\frac{1}{2\beta x}(x-\alpha)^2\right)}, x > 0 \tag{8}$$

The expected value and variance of the random variable are equal to:

$$E[X] = \alpha \quad Var(X) = \alpha\beta \tag{9}$$

With the imposed assumption that an unknown parameter of the Poisson distribution Θ is distributed according to the inverse Gaussian distribution – $\Theta: IG(\alpha, \beta)$, we assume that $E(\Theta) = 1$, since we need to find the average frequency of claims in one portfolio. Therefore, as for $\Theta: IG(1, \beta)$, and:

$$f_\Theta(\theta) = \frac{1}{\sqrt{2\pi\beta\theta^3}} e^{\left(-\frac{1}{2\beta\theta}(\theta-1)^2\right)} \tag{10}$$

we get an expression that represents the probability mass function, apropos, the resulting distribution of the number of claims in the portfolio:

$$p_k = \int_0^\infty e^{-\theta} \cdot \frac{\theta^k}{k!} \cdot \frac{1}{\sqrt{2\pi\beta\theta^3}} e^{\left(-\frac{1}{2\beta\theta}(\theta-1)^2\right)} d\theta, k = 0,1,2,\dots \quad (11)$$

An inverse Gaussian distribution is an excellent choice for modelling positive right-skewed data, which are typical for the frequency of claims arising from automobile liability insurance.

Research results and discussion

In order to determine which of the analysed models would be optimal for modelling the frequency of claims in automobile liability insurance, and why the Poisson model should be rejected, we need to test in practice the adaptability of the empirical frequency distribution. Testing shall be conducted by using the χ^2 goodness-of-fit test, which is the most widely used statistical test in the above mentioned sense [1, pp. 11-32]. Implementation of this test is based on the rule that all expected frequencies belong to five or more grouping procedures, with the level of significance $\alpha = 0.05$.

The sample size in the analysis is 98,978 automobile liability policyholders in one insurance company in Serbia. Table 1 shows the frequency distribution of the selected portfolio of claims in 2015.

Table 1: Observed distribution of the number of claims in the portfolio

Number of claims (X)	Number of policyholders (f)
0	88,928
1	9,235
2	755
3	55
4	5
≥ 5	0
Σ	98,978

The average (mean) number of claims in the sample is equal to:

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^n X_i \cdot f_i = 0.110429 \quad (12)$$

while the variance of the observed sample is equal to:

$$s^2 = \frac{1}{n-1} \cdot \sum_{i=1}^k f_i \cdot (X_i - \bar{x})^2 = 0.117431 \quad (13)$$

We shall now determine the expected frequency of claims by applying each of the considered models, and then test the goodness-of-fit for the obtained frequencies to the distribution assumed. Expected (theoretical) frequencies are obtained when the calculated probabilities (for each of the previous models) are multiplied by the sample size.

The information on the fitted Poisson distribution appear in Table 2.

Table 2: The number of observed and fitted claims for the Poisson model

Number of damages: X	Observed frequency (f _j)	Fitted frequency (f _i)
0	88,928	88,629.88
1	9,235	9,787.27
2	755	540.40
3	55	19.89
4	5	0.55
≥ 5	0	0.01

Using the maximum likelihood method or the method of moments, we can estimate the unknown Poisson distribution parameter, from which the following expected frequencies are derived:

$$f'_i = P(X = i|H_0) \cdot n = \frac{\hat{\theta}^i}{i!} \cdot e^{-\hat{\theta}} \cdot n, i = 1,2,3,4,5 \quad (14)$$

Table 3 presents the expected claim frequencies, under the assumption that the observed frequencies follow a negative binominal distribution. We have adjusted it according to the estimated parameters of negative binomial distribution by the moments method: $\hat{a} = \frac{\bar{x}^2}{s^2 - \bar{x}}$,

$$\hat{b} = \frac{\bar{x}}{s^2 - \bar{x}} \text{ and the recursion } P_{k+1} = P_k \cdot \frac{k + \hat{a}}{(k + a) \cdot (1 + \hat{b})}$$

Table 3: The number of observed and fitted claims for the negative binomial model

Number of damages: X	Observed frequency (f _j)	Fitted frequency (f _i)
0	88,928	88,928.19
1	9,235	9,234.60
2	755	754.82
3	55	56.14
4	5	3.97
≥5	0	0.27

Finally, we have used the Poisson-inverse Gaussian model and calculated the expected frequency of claims, where, for the estimation of the unknown distribution parameters $\hat{\alpha} = \bar{x}$, $\hat{\beta} = \frac{\bar{x}^2}{s^2 - \bar{x}}$ the method of moments is used, and the recursion method is used to determine the probability:

$$p_k = \frac{\beta(k-1)(2k-3)p_{k-1} + \alpha^2 p_{k-2}}{(1+2\beta)k(k-1)} \quad (15)$$

The results are illustrated in Table 4.

Table 4: The number of observed and fitted claims for the Poisson-inverse Gaussian model

Number of claims (X)	Observed frequency (f)	Fitted frequency (f')
0	88,928	88,922.45
1	9,235	9,250.46
2	755	741.45
3	55	58.41
4	5	4.78
≥5	0	0.41

Final conclusions derived are presented in Table 5, showing the obtained χ^2 statistics test values for each of the three models analysed.

Observed distribution adjustability to the assumed distribution will be tested with the following hypotheses:
 H_0 : The number of claims per auto insurance policy is adjusted to the assumed distribution
 H_1 : The observed distribution of the number of claims is not adjusted to the assumed distribution

The decision to accept an alternative or to retain the null hypothesis depends on the comparison between the obtained value of the χ^2 statistics from the above table and the corresponding critical value of the χ^2 distribution.

Let us evaluate the number of degrees of freedom for the χ^2 statistics. Since the class number of the analysed variable X _ number of claims equals $k = 5$ (the last expected

frequency does not equal 5 or more, so the total class number should be decreased by 1), and since the observed frequencies are dependable in a way that their number is fixed, we will have one degree of freedom less. Also, we had to estimate one parameter from the given data which was the reason for the loss of another degree of statistical freedom. Thus, the number of degrees of freedom, φ is equal to: $\varphi = 6 - 1 - 1 - 1 = 3$. From the χ^2 distribution, $\chi_{3,0.95}^2 = 7.815$, according to $\chi^2 = 215.44 > 7.815 = \chi_{3,0.95}^2$, we can conclude that the given distribution of claims does not comply with the Poisson distribution.

On the other hand, the number of degrees of freedom for the χ^2 statistics, in the case of the negative binomial model, as well as in case of the inverse Gaussian distribution, is equal to: $\varphi = 6 - 1 - 1 - 2 = 2$, because we had to estimate two parameters from the given data.

Using the critical value of the χ^2 distribution for an ascertained number of degrees of freedom suggests that the $\chi_{2,0.95}^2 = 5.991$, and since $\chi^2 = 0.53 < 5.991$ and $\chi^2 = 0.89 < 5.991$, we cannot reject the hypothesis of adjustability of the given distribution to a negative binomial distribution, as well as to a Poisson Inverse Gaussian distribution with a 5% risk of error.

Determining the probability of a distribution function, i.e. the model which best suits the observed frequency distribution of claims, shall be the beginning of the second stage in the process of establishing the premium rates.

In the observed case, it is determined that in the further process of establishing tariffs, the characteristics of two distributions could be applied. Since the approximation of the negative binomial distribution is slightly advanced (the distance of the realised χ^2 statistics from the critical value for this model is larger than in the case of the Poisson-inverse Gaussian model), in the text below we shall describe a model that establishes a system of optimal BMS, designed on the assumption of this distribution.

Table 5: Obtained χ^2 statistic test values and estimated parameters

Distribution	Poisson	Negative Binomial	Poisson Inverse Gaussian
Parameters	$\hat{\theta} = 0.110429$	$\hat{a} = 1.741346$ $\hat{b} = 15.768978$	$\hat{\alpha} = 0.110429$ $\hat{\beta} = 0.063416$
$\chi^2 = \frac{(f_i - f'_i)}{f'_i}$	215.44	0.53	0.89

One of the important characteristics of the negative binomial distribution, which will be of particular interest for this analysis, is that if the a priori distribution of the unknown Θ gamma, with the parameters a and b , in other words, if: $\Theta:\Gamma(a,b)$, than the a posteriori distribution of the claim frequency parameters is also gamma, whose parameters are now equal to:

$$a' = a + k \quad \text{and} \quad b' = b + t \quad (16)$$

where $k = \sum_{i=1}^t k_i$ represents the total number of claims per insured, and t represents the number of years to be taken into analysis.

Specifically, the estimate of the mean frequency of claims for the automobile portfolio, whose data on claims for the previous period are represented by the vector (k_1, k_2, \dots, k_t) , is equal to:

$$\theta_{t+1}(k_1, k_2, \dots, k_t) = \frac{a + k}{b + t} = \frac{a'}{b'} \quad (17)$$

Note that the stated expression represents a form of credibility theory according to which the premium is a weighted average of the individual risk and the average value of the collective risk.

In our case, assuming that the credibility factor can be represented in the form of: $z = \frac{t}{b + t}$, the estimate of the mean frequency will have the following form:

$$\theta_{t+1}(k_1, k_2, \dots, k_t) = z \cdot \frac{k}{t} + (1 - z) \cdot \frac{a}{b} \quad (18)$$

where $\frac{a}{b}$ represents the average, a priori premium, while $\frac{k}{t}$ represents the result of individual observations of the policyholder.

Finally, for the purpose of designing an optimal BMS for the given automobile portfolio based on the elements of the Bayesian analysis, we can apply a principle according to which we shall assign to each policyholder a premium proportional to the estimate of his/her claim frequency.

This simple principle, known as the expected value principle [3], [5], [6] requires the policyholder to pay a net premium plus a security loading α which is proportional to the net premium, and can be represented as:

$$P = (1 + \alpha)E(X), (\alpha > 0) \quad (19)$$

Thus, a policyholder, whose previous conduct in terms of reported claims is represented by the vector (k_1, k_2, \dots, k_t) , will pay a premium according to the following formula:

$$P(k_1, k_2, \dots, k_t) = c(1 + \alpha)\theta_{t+1}(k_1, k_2, \dots, k_t) = c(1 + \alpha)\frac{a'}{b'} \quad (20)$$

where c is a constant value, and α represents the security loading.

In accordance with this principle, the rule for establishing an optimal BMS for the analysed automobile portfolio may be illustrated in the form of the following ratio:

$$\frac{\int_0^{\infty} \theta du(\theta | k_1, k_2, \dots, k_t)}{\int_0^{\infty} \theta du(\theta)} \quad (21)$$

when, assuming that the amount of base (initial) premium is equal to 100 currency units, we reach an expression for determining a posteriori net premium, or a rule to determine the premium amount in the optimal BMS.

$$\begin{aligned} P'_{t+1}(k_1, k_2, \dots, k_t) &= 100 \cdot \frac{\frac{a + k}{b + t}}{\frac{a}{b}} \\ &= 100 \cdot \frac{b(a + k)}{a(t + b)} = 100 \cdot \frac{a'}{b'} \cdot \frac{b}{a} \end{aligned} \quad (22)$$

Following the results described above, in Table 6 we present the amounts of premiums that are supposed to be paid by the insured in the observed portfolio who is considered to cause k damages over a period of t years.

Data in Table 6 introduce a balance between the premium rates in such a way that a policyholder who reported no liability claim during a one-year period will gain a discount of nearly 6% in the next year, and an additional 5.6% in the following year without claims. Additional discount should be granted for up to 7 consecutive years without claims, in which case the discount would reach 30% of the nominal premium rate. On the other hand, filing one claim in a year will result in an increase in the premium price by 48%, while two claims will increase the nominal premium rate by nearly 100%.

Conclusions

Based on the elements of the credibility theory, by using the negative binomial distribution to model the frequency

Table 6: The optimal BMS – negative binominal distribution model

Number of years without claim		Number of claims reported by the insured					
t	0	1	2	3	4	5	6
0	100						
1	94.04	148.03	202.03	256.02	310.02	364.01	418.01
2	88.75	139.70	190.66	241.62	292.57	343.53	394.49
3	84.02	132.26	180.50	228.74	276.99	325.23	373.47
4	79.77	125.57	171.37	217.17	262.98	308.78	354.58
5	75.93	119.52	161.12	206.72	250.32	293.91	337.51
6	72.44	114.03	155.63	197.22	238.82	280.41	322.01
7	69.26	109.03	148.79	188.56	228.33	268.10	307.87

of claims and the expected value principle for determining the net premiums, we have managed to establish the tariff system. A suggested tariff model for auto liability premium rates is based on an optimal bonus-malus system. It complements the pioneering study of Lemaire [17] and it strongly supports the principle of fairness in the distribution of costs. By using such a model, it is possible to penalise drivers who are causing accidents, and this should result in reduction of claims and increase of safety in traffic.

Empirical research have proven that, in modelling the automobile liability rating system based on the frequency of claims, it is optimal to use sophisticated distribution models such as the negative binomial distribution. This facilitates the establishment of a financially balanced system of bonus-malus, which suggests the level of future premium rates for all expected risks which are equal to the cost of future accidents.

Taking into consideration the fact that the BMS in Serbia has been implemented merely a couple of years and that it is possible to improve its application, the results of this research paper could serve as a solid basis for developing and upgrading the process of determining premium rates in auto liability.

Finally, we would like to note that the established automobile liability tariffs that are derived in this paper, in other words – the presented optimal BMS, could be improved by introducing additional variables into the analysis. This primarily refers to several major a priori characteristics of the insured person, such as gender, age, and so forth, which have been discussed previously. Also, the process of establishing an optimal BMS can be further improved by including data on the cost amounts of claims

reported. Due to the fact that the data mentioned could not be implemented into the model presented, the authors of this paper intend to improve the tariff process as described above when the additional data become available.

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Ivana Simeunović

is Assistant Professor at the Faculty of Banking, Insurance and Finance, Union University. She teaches Financial Mathematics, Financial Statistics, Actuarial Mathematics, Insurance (undergraduate studies), Management Science, Quantitative Methods in Finance and Multivariate Analysis (postgraduate studies). Her scientific interests refer to: actuarial science (risk analysis and assessment, estimation of expected claims, loss reserving, premium calculation), statistical data analysis, financial mathematics. She published two books in the field of financial mathematics and financial statistics, as well as numerous papers related to the foregoing topics. She is a member of the Serbian Actuarial Association, Member of the Council of the Faculty and Head of the Actuarial Training which is held at the Belgrade Banking Academy.



Ivana Domazet

is Senior Research Fellow at the Institute of Economic Sciences and Associate Professor at the Faculty of Banking, Insurance and Finance, Union University. She teaches Marketing Principles (undergraduate studies), Marketing Management, Marketing Strategy and Integrated Communications (postgraduate studies). Her scientific interests refer to: marketing research and strategy, integrated marketing, communications, the CRM and strategic management. She published a book in the field of integrated marketing communications, as well as numerous papers related to the foregoing topics. She is President of the Scientific Council at the Institute of Economic Sciences, Vice-president of the Board of the said Institute, Board Member of the Institute of Social Sciences, member of the Scientific Society of Economists of Serbia and the Serbian Marketing Association.



Jovan Zubović

is Director and Senior Research Fellow at the Institute of Economic Sciences and Associate Professor at the Alfa BK University. He teaches the HRM and Strategic Management. His scientific interests refer to: labour economics, human resource management, evidence-based policies, policy impact evaluation and agricultural economics. He published a book in the field of impact of the FDI on human capital, edited three thematic monographs and published over 100 papers related to the foregoing topics. He is a member of the European Association of Labour Economists (EALE), European Association of Agricultural Economists (EAAE), and the National Association of Economists in Serbia.